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Sources of macroeconomic fluctuations: A regime-switching DSGE approach

Zheng Liu
Federal Reserve Bank of San Francisco

Daniel F. Waggoner
Federal Reserve Bank of Atlanta

Tao Zha
Federal Reserve Bank of Atlanta, Emory University, and NBER

We examine the sources of macroeconomic fluctuations by estimating a variety of richly parameterized DSGE models within a unified framework that incorporates regime switching both in shock variances and in the inflation target. We propose an efficient methodology for estimating regime-switching DSGE models. Our counterfactual exercises show that changes in the inflation target are not the main driving force of high inflation in the 1970s. The model that best fits the U.S. time-series data is the one with synchronized shifts in shock variances across two regimes, and the fit does not rely on strong nominal rigidities. We provide evidence that a shock to the capital depreciation rate, which resembles a financial shock, plays a crucial role in accounting for macroeconomic fluctuations.

Keywords. Regime switch, depreciation shock, financial shock, Müeller method, volatility changes, inflation target.

JEL classification. C11, C51, E32, E42, E52.

Zheng Liu: zheng.liu@sf.frb.org
Daniel F. Waggoner: Daniel.F.Waggoner@atl.frb.org
Tao Zha: tzha@frbatlanta.org

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1. Introduction

We examine the sources of macroeconomic fluctuations by estimating a number of regime-switching models using modern Bayesian techniques in a unified dynamic stochastic general equilibrium (DSGE) framework. The standard approach to analyzing business-cycle fluctuations is the use of constant-parameter medium-scale DSGE models (Christiano, Eichenbaum, and Evans (2005), Levin, Onatski, Williams, and Williams (2006), Del Negro, Schorfheide, Smets, and Wouters (2007), Smets and Wouters (2007), Altig, Christiano, Eichenbaum, and Linde (2011)). In this paper, we generalize the standard approach by allowing time variations in shock variances and in the central bank's inflation target according to Markov switching processes. These time variations appear to be present in the U.S. macroeconomic time series. An important question is how significant are the time variations when we fit the data to relatively large DSGE models with rich dynamic structures and shock processes that are economically interpretable? If the answer is positive, the next question is in what dimension do the time variations matter? To answer these questions, we estimate a number of alternative models nested in this general framework using the Bayesian method and we compare the fit of these models to the time-series data in the postwar U.S. economy. The best-fit model is then used to identify shocks that are important in driving macroeconomic fluctuations.

Our approach yields several new results. We find strong empirical evidence in favor of the DSGE model with two regimes in shock variances, where regime shifts in the variances are synchronized. The models with constant parameters (i.e., no regime shifts), with independent regime shifts in shock variances, or with more than two regimes do not fit to the data as well. In our preferred model (i.e., the best-fit model) with two synchronized shock regimes, the high-volatility regime was frequently observed in the period from the early 1970s through the mid-1980s, while the low-volatility regime prevailed in most of the period from the mid-1980s through 2007. This finding is broadly consistent with the well known fact that the U.S. economy experienced a general reduction in macroeconomic volatilities during the latter sample period (Stock and Watson (2003)).

The fit of our preferred regime-switching DSGE model does not rely on strong nominal rigidities. In particular, our estimates imply that the durations of the price and nominal wage contracts last no more than two quarters of a year—much shorter than those reported in the constant-parameter DSGE models in previous studies (Smets and Wouters (2007)). This finding highlights the sensitivity of the estimates of some key structural parameters obtained in models with no regime switching to specifications of the shock processes. When we allow the shock variances to switch regimes, the model relies less on nominal rigidities to fit to the data.

In addition, the fit of our preferred model does rely on regime shifts in the inflation target. Allowing the inflation target to shift between two regimes—either synchronized with or independent of the shock regime switching—does not improve the model's marginal data density. This finding is robust to a variety of model specifications and it is consistent with the conclusion from other works about changes in monetary policy in general (Stock and Watson (2003), Canova and Gambetti (2004), Cogley and Sargent (2005), Primiceri (2005), Sims and Zha (2006), Justiniano and Primiceri (2008)).

Although a change in the inflation target is not favored by the data, we find that, in an estimated DSGE model with regime switching in the inflation target, there are two distinct inflation-target regimes with a high target of about 5% of annualized inflation and a low target of about 2%. The probability of the high-target regime is near 1 in the first wave of great inflation in the early 1970s, but the probability stays near 0 in all other periods. This finding corroborates those in Sargent, Williams, and Zha (2006) and Carboni and Ellison (2009), who emphasized the role of the government's misconception about the unemployment–inflation trade-off in leading to the high inflation policy in the early 1970s. In our regime-switching framework, however, all agents are fully rational. Although the high-target regime was short lived (with the ergodic probability of about 3.5%), our counterfactual experiments show that the existence of the high-target regime and the probability of switching to that regime are important for explaining high inflation rates observed in the early 1970s, but not in other periods.

In our best-fit model with two synchronized regimes in shock variances, we identify three types of shocks that are important for macroeconomic fluctuations: a shock to the growth rate of the total factor productivity (TFP), a shock to wage markups, and a shock to the capital depreciation rate. Taken together, these three shocks account for about 70%–80% of the variances of aggregate output, investment, and inflation at business-cycle frequencies. Other shocks such as monetary policy shocks, investment-specific technology shocks, and price-markup shocks are not as important. The TFP shocks and the wage-markup shocks are fairly standard and well understood in the DSGE literature, but the capital depreciation shock is new. We provide some economic interpretations of a depreciation shock in Section 8.3.

In what follows, we briefly discuss our contributions in relation to the literature in Section 2. We then present, in Section 3, the general regime-switching DSGE framework. In Section 4, we present the system of equilibrium conditions and discuss our solution methods. In Section 5, we describe the data and our empirical approach. As a methodological contribution, we propose an efficient methodology for estimating regime-switching DSGE models; we summarize and discuss several modern methods for obtaining accurate estimates of marginal data densities for relatively large DSGE models. In Section 6, we compare the fit of a number of models nested by our general DSGE framework, identify the best-fit model, and report posterior estimates of the parameters in this model. In Section 7, we assess the importance of changes in the inflation-target regimes for explaining the high inflation rates observed in the 1970s based on a version of the DSGE model with two inflation-target regimes and with constant shock variances. In Section 8, we discuss the economic implications of our estimates in the best-fit model and identify the key sources of shocks that drive macroeconomic fluctuations. We conclude in Section 9. We describe the data used for our estimation in Appendix A. We also provide online technical appendices that describe the derivation of the equilibrium conditions, the log-linearized equations, and some details of the solution and estimation methods.[1]

[1]The online appendices are available in a supplementary file on the journal website at http://qeconomics.org/supp/71/supplement.pdf.

2. Related literature

The debate in the literature on the sources of macroeconomic fluctuations concerns whether shifts in monetary policy are the main sources of macroeconomic volatilities (Clarida, Galí, and Gertler (2000), Stock and Watson (2003), Lubik and Schorfheide (2004), Sims and Zha (2006), Bianchi (2008), Gambetti, Pappa, and Canova (2008)) or whether shocks in investment-specific technology are more important than other shocks in driving macroeconomic fluctuations (Fisher (2006), Smets and Wouters (2007), Justiniano and Primiceri (2008)). Much of the disagreement stems from the use of different frameworks and different empirical methods. Part of the literature focuses on reduced-form econometric models, part focuses on small-scale DSGE models, and part focuses on medium-scale DSGE models. Some models assume homogeneity in shock variances; others assume that shock variances are time-varying. Some models are estimated with different subsamples to reflect shifts in policy or in shock variances; other models are estimated with the entire sample. Given these differences in the model framework and in the empirical approach, it is difficult to draw a firm conclusion about the sources of macroeconomic fluctuations. The goal of the current paper is to provide a systematic examination of the sources of macroeconomic fluctuations in one unified DSGE framework that allows for regime shifts in shock variances and in monetary policy.

Our approach differs from that employed in the literature in several aspects. First, we aim to fully characterize the uncertainty across different models by examining different versions of the DSGE model for robust analysis to substantiate our conclusion. Although estimating a large set of models has not been performed in the literature, we think it is necessary to examine the robustness of a conclusion like ours about potential sources of macroeconomic fluctuations.

Second, our approach does not require splitting the sample to examine changes in monetary policy, although it nests sampling splitting as a special case. Unlike Sims and Zha (2006), where the number of vector autoregression (VAR) parameters is relatively large and the inflation target is implicit, our way of modeling policy changes takes the inflation target explicitly and gives a tightly parameterized model that has the best chance to detect the importance of policy changes, if they exist, in generating business-cycle fluctuations.

Third and methodologically, for fairly large DSGE models, especially for regime-switching DSGE models, the posterior distribution tends to be very non-Gaussian, making it very challenging to search for the global peak. We improve on earlier works such as Cogley and Sargent (2005) and Justiniano and Primiceri (2008) by obtaining the estimate of parameters at the posterior mode for each model. We show that economic implications can be seriously distorted if the estimates are based on a lower posterior peak.

Fourth, there is a strand of literature that emphasizes the importance of changes in the inflation target (for example, Erceg and Levin (2003), Favero and Rovelli (2003), Ireland (2005), Schorfheide (2005)). We follow this literature and focus on studying changes in the inflation target instead of changes in coefficients in the monetary policy rule for both conceptual and computational reasons. When agents take into account changes in monetary policy's response to inflation in forming their expectations, a solution method to the model is nonstandard (Liu, Waggoner, and Zha (2009)). Indeed, it

would be computationally infeasible for us to estimate a large set of DSGE models like we do in the current paper, since the solution would require an iterative algorithm that can be time-consuming in Monte Carlo Markov chain (MCMC) simulations. Furthermore, indeterminacy is more prevalent in this kind of regime-switching model than in the standard DSGE model (Farmer, Waggoner, and Zha (2009)). By modeling shifts in monetary policy as changes in the inflation target, we can apply the standard method to solve the DSGE models (as shown in Section 5). We have nonetheless pushed the limits of our computational and analytical capacity because of the large set of regime-switching models that we estimated.

Finally, we use three new methods for computing marginal data densities in model comparison. Since these methods are based on different statistical foundations, it is essential that all of these methods give a numerically similar result to ensure that the estimate of a marginal data density is unbiased and accurate (Sims, Waggoner, and Zha (2008)).

3. The model

The model economy is populated by a continuum of households, each endowed with a unit of differentiated labor skill indexed by $i \in [0, 1]$, and by a continuum of firms, each producing a differentiated good indexed by $j \in [0, 1]$. The monetary authority follows a feedback interest-rate rule, under which the nominal interest rate is set to respond to its own lag and deviations of inflation and output from their targets. The policy regime s_t, which is represented by the time-varying inflation target, switches between a finite number of regimes contained in the set $\mathcal{S}$, with the Markov transition probabilities summarized by the matrix $Q = [q_{ij}]$, where $q_{ij} = \text{Prob}(s_{t+1} = i | s_t = j)$ for $i, j \in \mathcal{S}$. The economy is buffeted by several sources of shocks. The variance of each shock switches between a finite number of regimes denoted by $s_t^* \in \mathcal{S}^*$, with the transition matrix $Q^* = [q_{ij}^*]$.

3.1 *The aggregation sector*

The aggregation sector produces a composite labor skill, denoted by L_t, to be used in the production of each type of intermediate good and a composite final good, denoted by Y_t, to be consumed by each household. The production of the composite skill requires a continuum of differentiated labor skills $\{L_t(i)\}_{i\in[0,1]}$ as inputs, and the production of the composite final good requires a continuum of differentiated intermediate goods $\{Y_t(j)\}_{j\in[0,1]}$ as inputs. The aggregation technologies are given by

$$L_t = \left[\int_0^1 L_t(i)^{1/\mu_{wt}}\, di\right]^{\mu_{wt}}, \qquad Y_t = \left[\int_0^1 Y_t(j)^{1/\mu_{pt}}\, dj\right]^{\mu_{pt}}, \tag{1}$$

where μ_{wt} and μ_{pt} determine the elasticity of substitution between the skills and between the goods, respectively. Following Smets and Wouters (2007), we assume that

$$\ln \mu_{wt} = (1 - \rho_w) \ln \mu_w + \rho_w \ln \mu_{w,t-1} + \sigma_{wt} \varepsilon_{wt} - \phi_w \sigma_{w,t-1} \varepsilon_{w,t-1} \tag{2}$$

and that

$$\ln \mu_{pt} = (1-\rho_p)\ln \mu_p + \rho_p \ln \mu_{p,t-1} + \sigma_{pt}\varepsilon_{pt} - \phi_p \sigma_{p,t-1}\varepsilon_{p,t-1}, \tag{3}$$

where, for $j \in \{w, p\}$, $\rho_j \in (-1, 1)$ is the (autoregressive) AR(1) coefficient, ϕ_j is the (moving average) MA(1) coefficient, $\sigma_{jt} \equiv \sigma_j(s_t^*)$ is the regime-switching standard deviation, and ε_{jt} is an independent and identically distributed (i.i.d.) white noise process with a zero mean and a unit variance. We interpret μ_{wt} and μ_{pt} as the wage-markup and price-markup shocks.

The representative firm in the aggregation sector faces perfectly competitive markets for the composite skill and the composite good. The demand functions for labor skill i and for good j resulting from the optimizing behavior in the aggregation sector are given by

$$L_t^d(i) = \left[\frac{W_t(i)}{\bar{W}_t}\right]^{-\mu_{wt}/(\mu_{wt}-1)} L_t, \qquad Y_t^d(j) = \left[\frac{P_t(j)}{\bar{P}_t}\right]^{-\mu_{pt}/(\mu_{pt}-1)} Y_t, \tag{4}$$

where the wage rate $\bar{W}_t$ of the composite skill is related to the wage rates $\{W_t(i)\}_{i\in[0,1]}$ of the differentiated skills by $\bar{W}_t = [\int_0^1 W_t(i)^{1/(1-\mu_{wt})}\,di]^{1-\mu_{wt}}$ and the price $\bar{P}_t$ of the composite good is related to the prices $\{P_t(j)\}_{j\in[0,1]}$ of the differentiated goods by $\bar{P}_t = [\int_0^1 P_t(j)^{1/(1-\mu_{pt})}\,dj]^{1-\mu_{pt}}$.

3.2 *The intermediate-good sector*

The production of a type j good requires labor and capital inputs. The production function is given by

$$Y_t(j) = K_t^f(j)^{\alpha_1}[Z_t L_t^f(j)]^{\alpha_2}, \tag{5}$$

where $K_t^f(j)$ and $L_t^f(j)$ are the inputs of capital and composite skill, and the variable Z_t denotes a neutral technology shock, which follows the stochastic process

$$Z_t = \lambda_z^t z_t, \quad \ln z_t = (1-\rho_z)\ln z + \rho_z \ln z_{t-1} + \sigma_{zt}\varepsilon_{zt}, \tag{6}$$

where $\rho_z \in (-1, 1)$ measures the persistence, $\sigma_{zt} \equiv \sigma_z(s_t^*)$ denotes the regime-switching standard deviation, and ε_{zt} is an i.i.d. white noise process with a zero mean and a unit variance. The parameters α_1 and α_2 measure the cost shares of the capital and labor inputs. Following Chari, Kehoe, and McGrattan (2000), we introduce some real rigidity by assuming the existence of some firm-specific factors (such as land), so that $\alpha_1 + \alpha_2 \le 1$.

Each firm in the intermediate-good sector is a price-taker in the input market and a monopolistic competitor in the product market where it sets a price for its product, taking the demand schedule in (4) as given. We follow Calvo (1983) and assume that pricing decisions are staggered across firms. Once a price is set, the firm has no other choice but to supply its differentiated product to meet market demand at that price. The probability that a firm cannot adjust its price is given by ξ_p. Following Woodford (2003), Christiano, Eichenbaum, and Evans (2005), and Smets and Wouters (2007), we

allow a fraction of firms that cannot reoptimize their pricing decisions to index their prices to the overall price inflation realized in the past period. Specifically, if firm j cannot set a new price, its price is automatically updated according to

$$P_t(j) = \pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p} P_{t-1}(j), \tag{7}$$

where $\pi_t = \bar{P}_t / \bar{P}_{t-1}$ is the inflation rate between $t-1$ and t, π is the steady-state inflation rate, and γ_p measures the degree of indexation.

A firm that can renew its price contract chooses $P_t(j)$ to maximize its expected discounted dividend flows given by

$$\mathrm{E}_t \sum_{i=0}^{\infty} \xi_p^i D_{t,t+i} [P_t(j) \chi_{t,t+i}^p Y_{t+i}^d(j) - V_{t+i}(j)], \tag{8}$$

where $D_{t,t+i}$ is the period-t present value of a dollar in a future state in period $t+i$, $V_{t+i}(j)$ is the cost function, and the term $\chi_{t,t+i}^p$ comes from the price-updating rule (7) and is given by

$$\chi_{t,t+i}^p = \begin{cases} \displaystyle\prod_{k=1}^{i} \pi_{t+k-1}^{\gamma_p} \pi^{1-\gamma_p}, & \text{if } i \geq 1, \\ 1, & \text{if } i = 0. \end{cases} \tag{9}$$

In maximizing its profit, the firm takes as given the demand schedule

$$Y_{t+i}^d(j) = \left(\frac{P_t(j) \chi_{t,t+i}^p}{\bar{P}_{t+i}} \right)^{-\mu_{p,t+i}/(\mu_{p,t+i}-1)} Y_{t+i}.$$

The first order condition for the profit-maximizing problem yields the optimal pricing rule

$$\mathrm{E}_t \sum_{i=0}^{\infty} \xi_p^i D_{t,t+i} Y_{t+i}^d(j) \frac{1}{\mu_{p,t+i} - 1} [\mu_{p,t+i} \Phi_{t+i}(j) - P_t(j) \chi_{t,t+i}^p] = 0, \tag{10}$$

where $\Phi_{t+i}(j) = \partial V_{t+i}(j) / \partial Y_{t+i}^d(j)$ denotes the marginal cost function. In the absence of markup shocks, μ_{pt} would be a constant and (10) implies that the optimal price is a markup over an average of the marginal costs for the periods in which the price will remain effective. Clearly, if $\xi_p = 0$ for all t, that is, if prices are perfectly flexible, then the optimal price would be a markup over the contemporaneous marginal cost.

Cost-minimizing implies that the marginal cost function is given by

$$\Phi_t(j) = \left[\tilde{\alpha} (\bar{P}_t r_{kt})^{\alpha_1} \left(\frac{\bar{W}_t}{Z_t} \right)^{\alpha_2} \right]^{1/(\alpha_1+\alpha_2)} Y_t(j)^{1/(\alpha_1+\alpha_2)-1}, \tag{11}$$

where $\tilde{\alpha} \equiv \alpha_1^{-\alpha_1} \alpha_2^{-\alpha_2}$ and r_{kt} denotes the real rental rate of capital input. The conditional factor demand functions imply that

$$\frac{\bar{W}_t}{\bar{P}_t r_{kt}} = \frac{\alpha_2}{\alpha_1} \frac{K_t^f(j)}{L_t^f(j)} \quad \forall j \in [0, 1]. \tag{12}$$

3.3 *Households*

There is a continuum of households, each endowed with a differentiated labor skill indexed by $h \in [0, 1]$. Household h derives utility from consumption and leisure. We follow Blanchard and Kiyotaki (1987) and assume that each household is a price-taker in the goods market and a monopolistic competitor in the labor market, where the householder sets a nominal wage for his or her differentiated labor skill, taking as given the wage index and the downward-sloping labor demand schedule in (4). The nominal wage-setting decisions by households are staggered in the spirit of Calvo (1983). Once a nominal wage rate is set, the household has to supply labor to meet the market demand for his or her differentiated skill at that wage rate (so quitting his or her job is not an option). We assume that there exist financial instruments that provide perfect insurance for the households in different wage-setting cohorts, so that the households make identical consumption and investment decisions despite the fact that their wage incomes may differ due to staggered wage setting.[2] In what follows, we impose this assumption and omit the household index for consumption and investment.

The utility function for household $h \in [0, 1]$ is given by

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t A_t\left\{\ln(C_t - bC_{t-1}) - \frac{\Psi}{1+\eta}L_t(h)^{1+\eta}\right\}, \tag{13}$$

where $\beta \in (0, 1)$ is a subjective discount factor, C_t denotes consumption, $L_t(h)$ denotes hours worked, $\eta > 0$ is the inverse Frisch elasticity of labor hours, and b measures the importance of habit formation. The variable A_t denotes a preference shock, which follows the stationary process

$$\ln A_t = (1 - \rho_a)\ln A + \rho_a \ln A_{t-1} + \sigma_{at}\varepsilon_{at}, \tag{14}$$

where $\rho_a \in (-1, 1)$ is the persistence parameter, $\sigma_{at} \equiv \sigma_a(s_t^*)$ is the regime-switching standard deviation, and ε_{at} is an i.i.d. white noise process with a zero mean and a unit variance.

In each period t, the household faces the budget constraint

$$\begin{aligned}&\bar{P}_t C_t + \frac{\bar{P}_t}{Q_t}[I_t + a(u_t)K_{t-1}] + \mathrm{E}_t D_{t,t+1}B_{t+1}\\ &\quad\le W_t(h)L_t^d(h) + \bar{P}_t r_{kt}u_t K_{t-1} + \Pi_t + B_t - T_t.\end{aligned} \tag{15}$$

In the budget constraint, I_t denotes investment, B_{t+1} is a nominal state-contingent bond that represents a claim to 1 dollar in a particular event in period $t + 1$, and this claim costs $D_{t,t+1}$ dollars in period t; $W_t(h)$ is the nominal wage for h's labor skill, K_{t-1} is the

[2]Obtaining complete risk-sharing among households in different wage-setting cohorts does not rely on the existence of such (implicit) financial arrangements. As shown by Huang, Liu, and Phaneuf (2004), the same equilibrium dynamics can be obtained in a model with a representative household (and thus complete insurance) consisting of a large number of worker members. The workers supply their homogenous labor skill to a large number of employment agencies, who transform the homogenous skill into differentiated skills and set nominal wages in a staggered fashion.

beginning-of-period capital stock, u_t is the utilization rate of capital, Π_t is the profit share, and T_t is lump-sum taxes used by the government to finance exogenous government spending. The function $a(u_t)$ captures the cost of variable capital utilization. Following Christiano, Eichenbaum, and Evans (2005) and Altig et al. (2011), we assume that $a(u)$ is increasing and convex. The term Q_t denotes the investment-specific technological change. Following Greenwood, Hercowitz, and Krusell (1997), we assume that Q_t contains a deterministic trend and a stochastic component. In particular,

$$Q_t = \lambda_q^t q_t, \tag{16}$$

where λ_q is the growth rate of the investment-specific technological change and q_t is an investment-specific technology shock, which follows a stationary process given by

$$\ln q_t = (1-\rho_q)\ln q + \rho_q \ln q_{t-1} + \sigma_{qt}\varepsilon_{qt}, \tag{17}$$

where $\rho_q \in (-1,1)$ is the persistence parameter, $\sigma_{qt} \equiv \sigma_q(s_t^*)$ is the regime-switching standard deviation, and ε_{qt} is an i.i.d. white noise process with a zero mean and a unit variance. The importance of investment-specific technological change is also documented in Fisher (2006) and Fernandez-Villaverde and Rubio-Ramirez (2007).

The capital stock evolves according to the law of motion

$$K_t = (1-\delta_t)K_{t-1} + [1 - S(I_t/I_{t-1})]I_t, \tag{18}$$

where the function $S(\cdot)$ represents the adjustment cost in capital accumulation. We assume that $S(\cdot)$ is convex and satisfies $S(\lambda_q\lambda^*) = S'(\lambda_q\lambda^*) = 0$, where $\lambda^* = (\lambda_q^{\alpha_1}\lambda_z^{\alpha_2})^{1/(1-\alpha_1)}$ is the steady-state growth rate of output and consumption. The term δ_t denotes the depreciation rate of the capital stock and follows the stationary stochastic process

$$\ln \delta_t = (1-\rho_d)\ln\delta + \rho_d \ln \delta_{t-1} + \sigma_{dt}\varepsilon_{dt}, \tag{19}$$

where $\rho_e \in (-1,1)$ is the persistence parameter, $\sigma_{dt} \equiv \sigma_d(s_t^*)$ is the regime-switching standard deviation, and ε_{dt} is the white noise innovation with a zero mean and a unit variance. We introduce this time variation in the depreciation rate to capture time-varying economic obsolescence and exogenous changes in the quality of capital emphasized by Justiniano, Primiceri, and Tambalotti (2011) and Gertler and Kiyotaki (2010).

The household takes prices and all wages but its own as given, and chooses C_t, I_t, K_t, u_t, B_{t+1}, and $W_t(h)$ to maximize (13) subject to (15)–(18), the borrowing constraint $B_{t+1} \geq -\underline{B}$ for some large positive number $\underline{B}$, and the labor demand schedule $L_t^d(h)$ described in (4).

The wage-setting decisions are staggered across households. In each period, a fraction ξ_w of households cannot reoptimize their wage decisions and, among those who cannot reoptimize, a fraction γ_w of them index their nominal wages to the price inflation realized in the past period. In particular, if household h cannot set a new nominal wage, its wage is automatically updated according to

$$W_t(h) = \pi_{t-1}^{\gamma_w}\pi^{1-\gamma_w}\lambda_{t-1,t}^* W_{t-1}(h), \tag{20}$$

where $\lambda^*_{t-1,t} \equiv \frac{\lambda^*_t}{\lambda^*_{t-1}}$, with $\lambda^*_t \equiv (Q_t^{\alpha_1} Z_t^{\alpha_2})^{1/(1-\alpha_1)}$ denoting the trend growth rate of aggregate output (and the real wage). If a household $h \in [0,1]$ can reoptimize its nominal wage-setting decision, it chooses $W(h)$ to maximize the utility subject to the budget constraint (15) and the labor demand schedule in (4). The optimal wage-setting decision implies that

$$\mathrm{E}_t \sum_{i=0}^{\infty} \xi_w^i D_{t,t+i} L^d_{t+i}(h) \frac{1}{\mu_{w,t+i} - 1} [\mu_{w,t+i} \mathrm{MRS}_{t+i}(h) - W_t(h) \chi^w_{t,t+i}] = 0, \tag{21}$$

where $\mathrm{MRS}_t(h)$ denotes the marginal rate of substitution between leisure and income for household h, and $\chi^w_{t,t+i}$ is defined as

$$\chi^w_{t,t+i} \equiv \begin{cases} \prod_{k=1}^{i} \pi_{t+k-1}^{\gamma_w} \pi^{1-\gamma_w} \lambda^*_{t,t+i}, & \text{if } i \geq 1, \\ 1, & \text{if } i = 0, \end{cases} \tag{22}$$

where $\lambda^*_{t,t+i} \equiv \frac{\lambda^*_{t+i}}{\lambda^*_t}$. In the absence of wage-markup shocks, μ_{wt} would be a constant and (21) implies that the optimal wage is a constant markup over a weighted average of the marginal rate of substitution for the periods in which the nominal wage remains effective. If $\xi_w = 0$, then the nominal wage adjustments are flexible and (21) implies that the nominal wage is a markup over the contemporaneous marginal rate of substitution.

3.4 *The government and monetary policy*

The government follows Ricardian fiscal policy, with its spending financed by lump-sum taxes so that $\bar{P}_t G_t = T_t$, where G_t denotes the government spending in final consumption units. Denote by $\tilde{G}_t \equiv \frac{G_t}{\lambda^*_t}$ the detrended government spending, where

$$\lambda^*_t \equiv (Z_t^{\alpha_2} Q_t^{\alpha_1})^{1/(1-\alpha_1)}. \tag{23}$$

We assume that $\tilde{G}_t$ follows the stationary stochastic process

$$\ln \tilde{G}_t = (1-\rho_g) \ln \tilde{G} + \rho_g \ln \tilde{G}_{t-1} + \sigma_{gt} \varepsilon_{gt} + \rho_{gz} \sigma_{zt} \varepsilon_{zt}, \tag{24}$$

where we follow Smets and Wouters (2007) and assume that the government spending shock responds to productivity shocks.

Monetary policy is described by a feedback interest-rate rule that allows the possibility of regime switching in the inflation target. The interest-rate rule is given by

$$R_t = \kappa R_{t-1}^{\rho_r} \left[r \pi \left(\frac{\pi_t}{\pi^*(s_t)} \right)^{\phi_\pi} \left(\frac{Y_t}{\lambda^*_t} \right)^{\phi_y} \right]^{1-\rho_r} e^{\sigma_{rt} \varepsilon_{rt}}, \tag{25}$$

where $R_t = [\mathrm{E}_t D_{t,t+1}]^{-1}$ denotes the nominal interest rate, $\pi^*(s_t)$ denotes the regime-dependent inflation target, $r = \lambda^*/\beta$ denotes the steady-state real interest rate, and π denotes the steady-state inflation rate. The constant terms ρ_r, ϕ_π, and ϕ_y are policy

parameters. The term κ is a constant that captures the steady-state value of detrended output in the Taylor rule. In particular, $\kappa = \tilde{Y}^{-\phi_y(1-\rho_r)}$, where $\tilde{Y}$ is the steady-state detrended output. The term ε_{rt} denotes the monetary policy shock, which follows an i.i.d. normal process with a zero mean and a unit variance. The term $\sigma_{rt} \equiv \sigma_r(s_t^*)$ is the regime-switching standard deviation of the monetary policy shock. We assume that the eight shocks ε_{wt}, ε_{pt}, ε_{zt}, ε_{qt}, ε_{dt}, ε_{at}, ε_{rt}, and ε_{gt} are mutually independent.

3.5 *Market clearing and equilibrium*

In equilibrium, markets for bond, composite labor, capital stock, and composite goods all clear. Bond market clearing implies that $B_t = 0$ for all t. Labor market clearing implies that $\int_0^1 L_t^f(j)\,dj = L_t$. Capital market clearing implies that $\int_0^1 K_t^f(j)\,dj = u_t K_{t-1}$. Composite goods market clearing implies that

$$C_t + \frac{1}{Q_t}[I_t + a(u_t)K_{t-1}] + G_t = Y_t, \tag{26}$$

where aggregate output is related to aggregate primary factors through the aggregate production function

$$G_{pt} Y_t = (u_t K_{t-1})^{\alpha_1} (Z_t L_t)^{\alpha_2}, \tag{27}$$

with $G_{pt} \equiv \int_0^1 (\frac{P_t(j)}{\bar{P}_t})^{(-\mu_{pt}/(\mu_{pt}-1))(1/(\alpha_1+\alpha_2))}\,dj$ measuring the price dispersion.

Given fiscal and monetary policy, an *equilibrium* in this economy consists of prices and allocations such that (i) taking prices and all nominal wages but its own as given, each household's allocation and nominal wage solve its utility maximization problem; (ii) taking wages and all prices but its own as given, each firm's allocation and price solve its profit maximization problem; (iii) markets clear for bond, composite labor, capital stock, and final goods.

4. Equilibrium dynamics

4.1 *Stationary equilibrium and the deterministic steady state*

We focus on a stationary equilibrium with balanced growth. On a balanced growth path, output, consumption, investment, capital stock, and the real wage all grow at constant rates, while hours remain constant. Further, in the presence of investment-specific technological change, investment and capital grow at a faster rate. To induce stationarity, we transform variables so that

$$\tilde{Y}_t = \frac{Y_t}{\lambda_t^*}, \qquad \tilde{C}_t = \frac{C_t}{\lambda_t^*}, \qquad \tilde{w}_t = \frac{W_t}{\bar{P}_t \lambda_t^*}, \qquad \tilde{I}_t = \frac{I_t}{Q_t \lambda_t^*}, \qquad \tilde{K}_t = \frac{K_t}{Q_t \lambda_t^*},$$

where λ_t^* is the underlying trend for output, consumption, and the real wage given by (23).

Along the balanced growth path, as noted by Greenwood, Hercowitz, and Krusell (1997), the real rental price of capital keeps falling since the capital–output ratio keeps

rising. The rate at which the rental price is falling is given by λ_q. Thus, the transformed variable $\tilde{r}_{kt} = r_{kt} Q_t$, that is, the rental price in consumption units, is stationary. Further, the marginal utility of consumption is declining, so we define $\tilde{U}_{ct} = U_{ct} \lambda_t^*$ to induce stationarity.

The steady state in the model is the stationary equilibrium in which all shocks are shut off, including the "regime shocks" to the inflation target. To derive the steady state, we represent the finite Markov switching process with a vector AR(1) process (Hamilton (1994)). Specifically, the inflation target can be written as

$$\pi^*(s_t) = [\pi^*(1), \pi^*(2)] e_{s_t}, \tag{28}$$

where $\pi^*(j)$ is the inflation target in regime $j \in \{1, 2\}$ and

$$e_{s_t} = \begin{bmatrix} \mathbf{1}\{s_t = 1\} \\ \mathbf{1}\{s_t = 2\} \end{bmatrix}, \tag{29}$$

with $\mathbf{1}\{s_t = j\} = 1$ if $s_t = j$ and $= 0$ otherwise. As shown in Hamilton (1994), the random vector e_{s_t} follows an AR(1) process

$$e_{s_t} = Q e_{s_{t-1}} + v_t, \tag{30}$$

where Q is the transition matrix of the Markov switching process and the innovation vector has the property that $\mathrm{E}_{t-1} v_t = 0$. In the steady state, $v_t = 0$ so that (30) defines the ergodic probabilities for the Markov process and, from (28), the steady-state inflation π is the ergodic mean of the inflation target such that the steady-state nominal interest rate is $R = r + \pi$. Given π, the derivations for the rest of the steady-state equilibrium conditions are straightforward.

4.2 *Linearized equilibrium dynamics*

To solve for the equilibrium dynamics, we log-linearize the equilibrium conditions around the deterministic steady state. We use a careted variable $\hat{x}_t$ to denote the log deviations of the stationary variable X_t from its steady-state value (i.e., $\hat{x}_t = \ln(X_t / X)$).

Linearizing the optimal pricing decision rule implies that

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \frac{\kappa_p}{1 + \bar{\alpha} \theta_p} (\hat{\mu}_{pt} + \hat{\mathrm{mc}}_t) + \beta \mathrm{E}_t [\hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t], \tag{31}$$

where $\theta_p \equiv \frac{\mu_p}{\mu_p - 1}$, $\kappa_p \equiv \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p}$, $\bar{\alpha} \equiv \frac{1 - \alpha_1 - \alpha_2}{\alpha_1 + \alpha_2}$, and

$$\hat{\mathrm{mc}}_t = \frac{1}{\alpha_1 + \alpha_2} [\alpha_1 \hat{r}_{kt} + \alpha_2 \hat{w}_t] + \bar{\alpha} \hat{y}_t. \tag{32}$$

This is the standard price–Phillips curve relation generalized to allow for partial dynamic indexation. In the special case without indexation (i.e., $\gamma_p = 0$), this relation reduces to the standard forward-looking Phillips curve relation, under which the price inflation depends on the current-period real marginal cost and the expected future inflation. In the presence of dynamic indexation, the price inflation also depends on its own lag.

Linearizing the optimal wage-setting decision rule implies that

$$\begin{aligned}&\hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t - \gamma_w \hat{\pi}_{t-1} \\ &\quad = \frac{\kappa_w}{1+\eta\theta_w}(\hat{\mu}_{wt} + \hat{\mathrm{mrs}}_t - \hat{w}_t) + \beta \mathrm{E}_t[\hat{w}_{t+1} - \hat{w}_t + \hat{\pi}_{t+1} - \gamma_w \hat{\pi}_t],\end{aligned} \tag{33}$$

where $\hat{w}_t$ denotes the log deviations of the real wage, $\hat{\mathrm{mrs}}_t = \eta \hat{l}_t - \hat{U}_{ct}$ denotes the marginal rate of substitution between leisure and consumption, $\theta_w \equiv \frac{\mu_w}{\mu_w - 1}$, and $\kappa_w \equiv \frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w}$ is a constant. To help understand the economics of this equation, we rewrite this relation in terms of the nominal wage inflation:

$$\begin{aligned}\hat{\pi}^w_t - \gamma_w \hat{\pi}_{t-1} = {} & \frac{\kappa_w}{1+\eta\theta_w}(\hat{\mu}_{wt} + \hat{\mathrm{mrs}}_t - \hat{w}_t) + \beta \mathrm{E}_t(\hat{\pi}^w_{t+1} - \gamma_w \hat{\pi}_t) \\ & + \frac{1}{1-\alpha_1}[\alpha_1(\Delta\hat{z}_t - \beta \mathrm{E}_t \Delta\hat{z}_{t+1}) + \alpha_2(\Delta\hat{q}_t - \beta \mathrm{E}_t \Delta\hat{q}_{t+1})].\end{aligned} \tag{34}$$

where $\hat{\pi}^w_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t + \Delta\hat{\lambda}^*_t$ denotes the nominal wage inflation. This nominal wage Phillips curve relation parallels that of the price–Phillips curve and has similar interpretations.

The rest of the linearized equilibrium conditions include

$$\begin{aligned}\hat{q}_{kt} = {} & S''(\lambda_I)\lambda_I^2 \Bigg\{ \Delta\hat{i}_t - \beta \mathrm{E}_t \Delta\hat{i}_{t+1} \\ & + \frac{1}{1-\alpha_1}[\alpha_2(\Delta\hat{z}_t - \beta \mathrm{E}_t \Delta\hat{z}_{t+1}) + \Delta\hat{q}_t - \beta \mathrm{E}_t \Delta\hat{q}_{t+1}] \Bigg\},\end{aligned} \tag{35}$$

$$\begin{aligned}\hat{q}_{kt} = {} & \mathrm{E}_t \Bigg\{ \Delta\hat{a}_{t+1} + \Delta\hat{U}_{c,t+1} - \frac{1}{1-\alpha_1}[\alpha_2 \Delta\hat{z}_{t+1} + \Delta\hat{q}_{t+1}] \\ & + \frac{\beta}{\lambda_I}[(1-\delta)\hat{q}_{k,t+1} - \delta\hat{\delta}_{t+1} + \tilde{r}_k \hat{r}_{k,t+1}] \Bigg\},\end{aligned} \tag{36}$$

$$\hat{r}_{kt} = \sigma_u \hat{u}_t, \tag{37}$$

$$0 = \mathrm{E}_t \left[\Delta\hat{a}_{t+1} + \Delta\hat{U}_{c,t+1} - \frac{1}{1-\alpha_1}[\alpha_2 \Delta\hat{z}_{t+1} + \alpha_1 \Delta\hat{q}_{t+1}] + \hat{R}_t - \hat{\pi}_{t+1} \right], \tag{38}$$

$$\hat{k}_t = \frac{1-\delta}{\lambda_I}\left[\hat{k}_{t-1} - \frac{1}{1-\alpha_1}(\alpha_2 \Delta\hat{z}_t + \Delta\hat{q}_t)\right] - \frac{\delta}{\lambda_I}\hat{\delta}_t + \left(1 - \frac{1-\delta}{\lambda_I}\right)\hat{i}_t, \tag{39}$$

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{i}_t + u_y \hat{u}_t + g_y \hat{g}_t, \tag{40}$$

$$\hat{y}_t = \alpha_1\left[\hat{k}_{t-1} + \hat{u}_t - \frac{1}{1-\alpha_1}(\alpha_2 \Delta\hat{z}_t + \Delta\hat{q}_t)\right] + \alpha_2 \hat{l}_t, \tag{41}$$

$$\hat{w}_t = \hat{r}_{kt} + \hat{k}_{t-1} + \hat{u}_t - \frac{1}{1-\alpha_1}(\alpha_2 \Delta\hat{z}_t + \Delta\hat{q}_t) - \hat{l}_t, \tag{42}$$

where (35) is the linearized investment decision equation with $\hat{q}_{kt}$ denoting the shadow value of existing capital (i.e., Tobin's Q) and Δ denoting the first-difference operator (so

that $\Delta x_t = x_t - x_{t-1}$); (36) is the linearized capital Euler equation; (37) is the linearized capacity utilization decision equation with $\sigma_u \equiv \frac{a''(1)}{a'(1)}$ denoting the curvature of the function $a(u)$ evaluated at the steady state; (38) is the linearized bond Euler equation; (39) is the linearized law of motion for the capital stock; (40) is the linearized aggregate resource constraint, with the steady-state ratios given by $c_y = \frac{\tilde{C}}{\tilde{Y}}$, $i_y = \frac{\tilde{I}}{\tilde{Y}}$, $u_y = \frac{\tilde{r}_k \tilde{K}}{\tilde{Y}\lambda_I}$, and $g_y = \frac{\tilde{G}}{\tilde{Y}}$; (41) is the linearized aggregate production function; and (42) is the linearized factor demand relation.

Finally, the linearized interest-rate rule is given by

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1-\rho_r)\big[\phi_\pi(\hat{\pi}_t - \hat{\pi}^*(s_t)) + \phi_y \hat{y}_t\big] + \sigma_{rt}\varepsilon_{rt}, \tag{43}$$

where the term $\hat{\pi}^*(s_t) \equiv \log \pi^*(s_t) - \log \pi$ denotes the deviations of the inflation target from its ergodic mean.

5. Estimation approach

We estimate the parameters in our model using the Bayesian method. We describe a general empirical strategy so that the method can be applied to other regime-switching DSGE models. As shown in the online independent appendices, our model contains 27 variables. Adding the 5 lagged variables $\hat{y}_{t-1}$, $\hat{c}_{t-1}$, $\hat{i}_{t-1}$, $\hat{w}_{t-1}$, and $\hat{q}_{t-1}$ to the list gives a total of 33 variables. We denote all these state variables by the vector f_t, where f_t is so arranged that the first 8 variables are $\hat{y}_t$, $\hat{c}_t$, $\hat{i}_t$, $\hat{w}_t$, $\hat{q}_t$, $\hat{\pi}_t$, $\hat{\ell}_t$, and $\hat{R}_t$ and the last 5 variables are $\hat{y}_{t-1}$, $\hat{c}_{t-1}$, $\hat{i}_{t-1}$, $\hat{w}_{t-1}$, and $\hat{q}_{t-1}$.

We apply the relation (28) to the policy rule (43), where the vector e_{s_t} defined in (29) follows a vector AR(1) process described in (30). Expanding the log-linearized system with the additional variables represented by e_{s_t} maintains the log-linear form in which all coefficients are constant (i.e., independent of regime changes). A standard solution technique, such as the method proposed by Sims (2002), can be directly utilized to solve our DSGE model. The solution leads to the following VAR(1) form of state equations[3]:

$$f_t = c(s_t) + F f_{t-1} + C(s_t^*)\varepsilon_t, \tag{44}$$

where $\varepsilon_t = [\varepsilon_{rt}, \varepsilon_{pt}, \varepsilon_{wt}, \varepsilon_{gt}, \varepsilon_{zt}, \varepsilon_{at}, \varepsilon_{dt}, \varepsilon_{qt}]'$, and $c(s_t)$ is a vector function of the inflation targets $\pi^*(s_t)$ and the elements in the transition matrix Q, and $C(s_t^*)$ is a matrix function of $\sigma_{rt}(s_t^*)$, $\sigma_{pt}(s_t^*)$, $\sigma_{wt}(s_t^*)$, $\sigma_{gt}(s_t^*)$, $\sigma_{zt}(s_t^*)$, $\sigma_{at}(s_t^*)$, $\sigma_{dt}(s_t^*)$, and $\sigma_{qt}(s_t^*)$.

It follows from (44) that the solution to our DSGE model depends on the composite regime (s_t, s_t^*). If s_t^* is assumed to be completely synchronized with s_t as in Schorfheide (2005), then the composite regime collapses to s_t. To simplify our notation and keep analytical expressions tractable, we use s_t for the rest of this section to represent a composite regime that may mean (s_t, s_t^*), as long as no confusion about the notation arises.

Our estimation is based on the 1959:I–2007:IV quarterly time-series observations on eight U.S. aggregate variables:[4] real per capita gross domestic product (GDP) (Y_t^{Data}),

[3]The reduced form from solving the DSGE model will, in general, have the constant term depending on both s_t and s_{t-1}, because the term v_t in (30), which has to be substituted out, depends on s_{t-1}. It can be shown, however, that all the terms involving s_{t-1} will be cancelled out.

[4]We did not include the sample after 2007 because it is beyond the scope of this paper to address the current financial crisis and the effect of monetary policy at the lower zero bound.

real per capita consumption (C_t^{Data}), real per capita investment (I_t^{Data}), real wage (w_t^{Data}), the investment-specific technology (i.e., the biased technology Q_t), the quarterly GDP-deflator inflation rate (π_t^{Data}), per capita hours (L_t^{Data}), and the (annualized) federal funds rate ($\text{FFR}_t^{\text{Data}}$). Note that I_t^{Data} corresponds to $\frac{I_t}{Q_t}$ in the model (i.e., investment measured in units of consumption goods); a detailed description of the data is in Appendix A. These data are represented by the vector of observable variables

$$y_t = \Bigg[\Delta \ln Y_t^{\text{Data}}, \Delta \ln C_t^{\text{Data}}, \Delta \ln I_t^{\text{Data}}, \Delta \ln w_t^{\text{Data}}, \ln \pi_t^{\text{Data}}, \Delta \ln Q_t^{\text{Data}}, \ln L_t^{\text{Data}}, \frac{\text{FFR}_t^{\text{Data}}}{400}\Bigg]'.$$

The observable vector is connected to the model (state) variables through the measurement equations

$$y_t = a + Hf_t,$$

where

$$a = [\ln \lambda_*, \ln \lambda_*, \ln \lambda_*, \ln \lambda_*, \ln \pi, \ln \lambda_q, \ln L, \ln R]'. \tag{45}$$

Given the aforementioned regime-switching state space form, the model can be estimated by following the general estimation methodology of Sims, Waggoner, and Zha (2008).

5.1 *Three methods for computing marginal data densities*

To evaluate the model's fit to the data and compare it to the fit of other models, it is desirable to compute the marginal data density implied by the model. To keep the notation simple, let θ represent a collection of all model parameters, including all free parameters in the transition matrix. The marginal data density is defined as

$$p(Y_T) = \int p(Y_T|\theta)p(\theta)\,d\theta, \tag{46}$$

where the likelihood function $p(Y_T|\theta)$ can be evaluated recursively. For many empirical models, the modified harmonic mean (MHM) method of Gelfand and Dey (1994) is widely used to compute the marginal data density. The MHM method used to approximate (46) numerically is based on a theorem that states

$$p(Y_T)^{-1} = \int_\Theta \frac{h(\theta)}{p(Y_T|\theta)p(\theta)} p(\theta|Y_T)\,d\theta, \tag{47}$$

where Θ is the support of the posterior probability density and $h(\theta)$, often called a *weighting* function, is any probability density whose support is contained in Θ. Denote

$$m(\theta) = \frac{h(\theta)}{p(Y_T|\theta)p(\theta)}.$$

A numerical evaluation of the integral on the right hand side of (47) can be accomplished, in principle, through the Monte Carlo (MC) integration

$$\hat{p}(Y_T)^{-1} = \frac{1}{N}\sum_{i=1}^{N} m\big(\theta^{(i)}\big), \tag{48}$$

where $\theta^{(i)}$ is the ith draw of θ from the posterior distribution represented by $p(\theta|Y_T)$. If $m(\theta)$ is bounded above, the rate of convergence from this MC approximation is likely to be practical.

Geweke (1999) proposed a Gaussian function for $h(\cdot)$ constructed from the posterior simulator. The likelihood and posterior density functions for our medium-scale DSGE model turn out to be quite non-Gaussian and there exist zeros of the posterior probability density function (pdf) in the interior points of the parameter space. In this case, the standard MHM procedure tends to be unreliable, because the MCMC draws are likely to be dominated by a few draws as the number of draws increases. Sims, Waggoner, and Zha (2008) proposed a truncated non-Gaussian weighting function for $h(\cdot)$ to remedy the problem. This weighting function seems to work well for the non-Gaussian posterior density.

In addition to the method of Sims, Waggoner, and Zha (2008), we use the unpublished method developed by Ulrich Müeller at Princeton University. The Müeller method also depends crucially on a good choice of the weighting function $h(\theta)$. To summarize the Müeller method for computing the marginal data density, we note that $p(\theta|Y_T)$ is the posterior pdf of *unknown* form. Let $p^*(\theta|Y_T)$ be the posterior kernel (*not* pdf) such that $p^*(\theta|Y_T) = p(Y_T|\theta)p(\theta)$. Thus, $p(\theta|Y_T) = c^* p^*(\theta|Y_T)$, where $c^* = p(Y_T)^{-1}$.

Given any parameter value of θ, we know how to calculate the likelihood function $p(Y_T|\theta)$, the prior $p(\theta)$, and the kernel $p^*(\theta|Y_T)$. Our objective is to obtain an accurate estimate of the marginal data density $p(Y_T)$, which amounts to computing the constant term c^*. To achieve this goal, we first choose the weighting pdf $h(\theta)$ and then construct the function $f(c)$ as

$$\begin{aligned} f(c) = {} & \mathrm{E}_h\left[\mathbf{1}\left\{\frac{cp^*(\theta|Y_T)}{h(\theta)} < 1\right\}\left(1 - \frac{cp^*(\theta|Y_T)}{h(\theta)}\right)\right] \\ & - \mathrm{E}_p\left[\mathbf{1}\left\{\frac{h(\theta)}{cp^*(\theta|Y_T)} < 1\right\}\left(1 - \frac{h(\theta)}{cp^*(\theta|Y_T)}\right)\right], \end{aligned}$$

where $\mathbf{1}\{x\}$ is an indicator function that returns to 1 if x is true and to 0 if false, c is a positive real number, E_h is a mathematical expectation operator with respect to the weighting distribution represented by $h(\theta)$, and E_p is a mathematical expectation operator with respect to the posterior distribution represented by $p(\theta|Y_T)$. It can be shown that the function has the following properties:

- $f(c)$ is monotonically decreasing in c.
- $f(0) = 1$ and $f(\infty) = -1$.

Given these properties, we can use a bisection method to find an estimate of c^* where $f(c^*) = 0$.

To implement the Müeller method, it is important to bear in mind that although we do not know the functional form of the posterior pdf $p(\theta|Y_T)$, we can draw from this distribution through MCMC simulations. Let $\theta^{(i)}$ be the ith draw of θ from the posterior distribution with a total of N_p draws and let $\theta^{(j)}$ be the jth draw of θ from the weighting distribution with a total of N_h draws. The function value $f(c)$ can be approximated through numerical integration. Denote this approximation by $\hat{f}(c)$, which can be computed as

$$\hat{f}(c) = \frac{1}{N_h}\sum_{j=1}^{N_h}\left[\mathbf{1}\left\{\frac{cp^*(\theta^{(j)}|Y_T)}{h(\theta^{(j)})} < 1\right\}\left(1 - \frac{cp^*(\theta^{(j)}|Y_T)}{h(\theta^{(j)})}\right)\right]$$
$$- \frac{1}{N_p}\sum_{i=1}^{N_p}\left[\mathbf{1}\left\{\frac{h(\theta^{(i)})}{cp^*(\theta^{(i)}|Y_T)} < 1\right\}\left(1 - \frac{h(\theta^{(i)})}{cp^*(\theta^{(i)}|Y_T)}\right)\right].$$

Posterior draws are computationally more expensive than draws from the weighting pdf. Once all these draws are made, however, the calculation of $\hat{f}(c)$ is trivial for any positive value of c. The estimate of $c^* = p(Y_T)^{-1}$ such that $\hat{f}(c^*) = 0$ can be found by using a bisection method.

A third method we use is the bridge sampling of Meng and Wong (1996). The bridge-sampling method is often regarded as one of the most reliable methods for computing the Bayes factor. Since the preceding three methods are developed from different mathematical relationships, we recommend using all three methods to ensure that the estimated value of the marginal data density is numerically similar across methods.

Because the posterior density function is very non-Gaussian and complicated in shape, it is all the more important to find the posterior mode via an optimization routine. The estimate of the mode not only represents the most likely value (and thus the posterior estimate), but also serves as a crucial starting point for initializing different chains of MCMC draws.

For various DSGE models studied in this paper, finding the mode has proven to be a computationally challenging task. The optimization method we use combines the blockwise BFGS algorithm developed by Sims, Waggoner, and Zha (2008) and various constrained optimization routines contained in the commercial International Mathematics and Statistics Library (IMSL) package. The blockwise Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm, following the idea of Gibbs sampling and the expectation-maximization (EM) algorithm, breaks the set of model parameters into subsets and uses Christopher A. Sims's `csminwel` program to maximize the likelihood of one set of the model's parameters conditional on the other sets.[5] Maximization is iterated at each subset until it converges. Then the optimization iterates between the blockwise BFGS algorithm and the IMSL routines until it converges. The convergence criterion is the square root of machine epsilon.

[5]The `csminwel` program can be found at http://sims.princeton.edu/yftp/optimize/.

Thus far we have described the optimization process for only one starting point.[6] Our experience is that without such a thorough search, one can be easily misled to a much lower posterior value (e.g., a few hundreds lower in log value than the posterior peak). We thus use a set of cluster computing tools described in Ramachandran, Urazov, Waggoner, and Zha (2007) to search for the posterior mode. We begin with a grid of 100 starting points; after convergence, we perturb each maximum point in both small and large steps to generate an additional 20 new starting points, and we restart the optimization process again; the posterior estimates attain the highest posterior density value. The other converged points typically have much lower likelihood values by at least a magnitude of hundreds of log values. For each DSGE model, the peak value of the posterior kernel and the mode estimates are reported.

5.2 *Priors*

We set three parameters a priori. We set the steady-state government spending to output ratio at $g_y = 0.18$. We follow Justiniano and Primiceri (2008) and fix the persistence of the government spending shock process at $\rho_g = 0.99$. As noted by Smets and Wouters (2007), all these government parameter are difficult to estimate unless government spending is included in the set of measurement equations. Finally, we normalize and fix the steady-state hours worked at $L = 0.2$. We estimate all the remaining parameters. Tables 3 and 4 summarize the prior distributions for the structural parameters and the shock parameters.

Our priors are chosen to be more flexible and less tight than those in the previous literature. Specifically, instead of specifying the mean and the standard deviation, we use the 90% probability interval to back out the hyperparameter values of the prior distribution.[7] The intervals are generally set wide enough to allow the possibility of multiple posterior peaks (Del Negro and Schorfheide (2008)). Our approach is also necessary to deal with skewed distributions and allow for some reasonable hyperparameter values in certain distributions (such as the inverse-gamma) where the first two moments may not exist. The probability intervals reported in Table 3 cover the calibrated value of each parameter.

We begin with the preference parameters b, η, and β. Our prior for the habit-persistence parameter b follows the beta distribution. We choose the two hyperparameters of the beta distribution such that the lower bound for b (0.05) has a cumulative probability of 5% and the upper bound (0.948) has a cumulative probability of 95%. This 90% probability interval for b covers the values used by most economists (for example, Boldrin, Christiano, and Fisher (2001) and Christiano, Eichenbaum, and Evans (2005)). Our prior for the inverse Frisch elasticity η follows the gamma distribution. We choose the two hyperparameters of the gamma distribution such that the lower bound (0.2) and

[6]For the no-switching (constant-parameter) DSGE model, it takes a couple of hours to find the posterior peak. While the model with two-regime shock variances takes about 20 hours to converge, the model with two-regime inflation targets and two-regime shock variances takes four times longer.

[7]The program for backing out the hyperparameter values of a given prior can be found at https://webdrive.service.emory.edu/users/tzha/ProgramCode/ProgramCode.html under Matlab library.

the upper bound (10.0) of η correspond to the 90% probability interval. This prior range for η implies that the Frisch elasticity lies between 0.1 and 5, a range broad enough to cover the values based on both microeconomic evidence (Pencavel (1986)) and macroeconomic studies (Rupert, Rogerson, and Wright (2000)). Our prior for the transformed subjective discount factor $\chi_\beta \equiv 100(\frac{1}{\beta} - 1)$ follows the gamma distribution, with the hyperparameters appropriately chosen such that the bounds for the 90% probability interval of χ_β are 0.2 and 4.0. The implied value of β lies in the range between 0.9615 and 0.998, which nests the values obtained by Smets and Wouters (2007) ($\beta = 0.9975$) and Algit et al. (2011) ($\beta = 0.9926$).

Next, we discuss the prior distributions for the technology parameters α_1, α_2, λ_q, λ_*, σ_u, S'', and δ. Our priors for the labor share and capital share both follow the beta distribution with the restriction $\alpha_1 + \alpha_2 \leq 1$, so that the production technology requires firm-specific factors (Chari, Kehoe, and McGrattan (2000)). Specifically, the bounds for the α_1 values in the 90% probability interval are 0.15 and 0.35; those for α_2 are 0.35 and 0.75. With the restriction $\alpha_1 + \alpha_2 \leq 1$, however, the joint 90% probability region would be somewhat different. We assume that the priors for the (transformed) trend growth rates of the investment-specific technology and the neutral technology both follow the gamma distribution, with the 5% and 95% bounds given by 0.1 and 1.5, respectively. These values imply that, with 90% probability, the prior values for the trend growth rates λ_q and λ_* lie in the range between 1.001 and 1.015 (corresponding to annual rates of 0.4% and 6%, respectively). We assume that the priors for the capacity utilization parameter σ_u and the investment adjustment cost parameter S'' both follow the gamma distribution, with the lower bounds given by 0.5 and 0.1 and the upper bounds given by 3.0 and 5.0, respectively. These 90% probability ranges cover the values obtained, for example, by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). We assume that the prior for the average annualized depreciation rate follows the beta distribution with the 90% probability range lying between 0.05 and 0.20.

Third, we discuss the prior distributions for the parameters that characterize price and nominal wage setting in the model. These include the average price markup μ_p, the average wage markup μ_w, the Calvo probabilities of nonadjustment in pricing ξ_p and in wage-setting ξ_w, and the indexation parameters γ_p and γ_w. The priors for the net markups $\mu_p - 1$ and $\mu_w - 1$ both follow the gamma distribution with the 90% probability range covering the values between 0.01 and 0.5. This range covers most of the calibrated values of the markup parameters used in the literature (e.g., Basu and Fernald (2002), Rotemberg and Woodford (1997), Huang and Liu (2002)). The priors for the price and wage duration parameters ξ_p and ξ_w both follow the beta distribution with the 90% probability range between 0.1 and 0.75. Under this prior distribution, the nominal contract durations vary, with 90% probability, between 1.1 quarters and 4 quarters. This range covers the values of the frequencies of price and wage adjustments used in the literature (e.g., Bils and Klenow (2004), Taylor (1999)). The priors for the indexation parameters γ_p and γ_w both follow the uniform distribution with the 90% probability range lying between 0.05 and 0.95. In this sense, we have loose priors on these indexation parameters, the range of which covers those used in most studies (e.g., Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Woodford (2003)).

Finally, we discuss the coefficients in the monetary policy rule, including ρ_r, ϕ_π, and ϕ_y. The prior for the interest-rate smoothing parameter ρ_r follows the beta distribution with the 90% probability range between 0.05 and 0.948. The prior for the inflation coefficient ϕ_π follows the gamma distribution with the 90% probability range between 0.5 and 5.0. The prior for the output coefficient ϕ_y follows the gamma distribution with the 90% probability range between 0.05 and 3.0. This range includes the values obtained by Clarida, Galí, and Gertler (2000) and others. These prior values allow for an indeterminate region. When the equilibrium is indeterminate, we follow Boivin and Giannoni (2006) and use the Minimum State Variable (MSV) solution. In our MCMC simulations, however, there is practically little probability for the parameters to be in the indeterminate region.

Our priors for the AR(1) coefficients for the neutral and biased technology shocks ρ_q and ρ_z are uniformly distributed in the $[0, 1]$ interval. The AR(1) coefficients for all other shocks and the MA(1) coefficients for the price- and wage-markup shocks follow the beta distribution with the 5%–95% probability range given by $[0.05, 0.948]$. The prior for the parameter ρ_{gz} follows the gamma distribution with the 90% probability range given by $[0.2, 3.0]$. The standard deviations of each of the eight shocks follow the inverse gamma distribution with the 90% probability range given by $[0.0005, 1.0]$. This probability range implies a more agnostic prior than was used by Smets and Wouters (2007) and Justiniano and Primiceri (2008). Such an agnostic prior is needed to allow for possible large changes in shock variances across regimes, as found in Sims and Zha (2006).

We have experimented with different priors. In one alternative prior, we follow the literature and make a prior on the persistence parameters in shock processes much tighter toward zero, such as the beta$(1, 2)$ probability density. Our conclusions hold true for these priors as well.

6. Empirical results

In this section, we report our main empirical findings. We compare in Section 6.1 the empirical fit of a variety of models nested by our general regime-switching DSGE framework. We then report in Section 6.2 the estimation results in our best-fit model and highlight the difference of these estimates from some alternative models.

6.1 *Model fit*

The first set of results we discuss is measures of model fit, with the comparison based on maximum log posterior densities adjusted by the Schwarz criterion.[8] Table 1 reports Schwarz criteria for different versions of our DSGE model (the column labeled Baseline) and for models with the restriction that all the persistence parameters in both price-markup and wage-markup processes are set to zero (the column labeled Restricted).

Table 1 shows that the model with regime shifts in shock variances only (DSGE-2v) is the best-fit model, much better than the constant-parameter DSGE model (DSGE-con). The Schwarz criterion for the baseline DSGE-2v model is 5963.03, compared to

[8]The Schwarz criterion is similar to the Laplace approximation used by Smets and Wouters (2007).

TABLE 1. Schwarz criterion for the set of DSGE models.

Model[a]	Baseline[b]	Restricted[c]
DSGE-con	5859.71	5811.14
DSGE-2v	5963.03	5920.01
DSGE-2c	5853.13	5805.27
DSGE-2cv	5960.71	5907.00
DSGE-2c2v	5958.78	5913.85
DSGE-2v2v	5958.18	5912.22
DSGE-3v	5950.73	5926.91

[a]The models studied are the DSGE model with all parameters that are constant across time (DSGE-con), the DSGE model with two regimes in shock variances (DSGE-2v), the DSGE model with two regimes in the inflation target only (DSGE-2c), the DSGE model with two common regimes for both shock variances and the inflation target (DSGE-2cv), and the DSGE model with two independent Markov processes, one controlling two regimes in shock variances and the other controlling two regimes in the inflation target (DSGE-2c2v), the DSGE model with two independent Markov processes, one controlling two regimes in variances of two technology shocks and the other controlling two regimes in variances of all the other shocks (DSGE-2v2v), and the DSGE model with three regimes in shock variances (DSGE-3v).

[b]The posterior densities at the posterior mode, adjusted by Schwarz criterion.

[c]The posterior densities evaluated at the posterior modes for models with the persistence parameters in both the price- and wage-markup processes set to zero.

5859.71 for the DSGE-con model. When we allow the inflation target to switch regimes while holding the shock variances constant (DSGE-2c), the model's fit does not improve on the constant-parameter DSGE model. When we allow both the inflation target and shock variances to switch regimes with the same Markov process (i.e., regime switching is synchronized), the model (DSGE-2cv) does better than the model with regime switching in the inflation target alone, but it does not improve on the baseline DSGE-2v model with regime shifts in the shock variances only. When we relax the assumption that switches in the shock regime and in the inflation target regime are synchronized, and we compute the Schwarz criterion for the model with the target regime and the shock regime independent of each other (DSGE-2c2v), we find that the model's fit does not improve relative to either the DSGE-2cv model with synchronized regime shifts in the inflation target and the shock variances or the baseline DSGE-2v model with synchronized regime shifts in shock variances only. We have also examined the possibility of three shock regimes instead of two. We find that the three-regime model (DSGE-3v) does not improve on the baseline two-regime model (DSGE-2v).

We have also estimated models with shock variances that follow independent Markov switching processes. This scenario approximates stochastic volatility models, where each shock variance has its own independent stochastic process (Tauchen (1986), Sims, Waggoner, and Zha (2008)). In addition, we have grouped a subset of shock variances that have the same Markov processes. None of these models fits to the data better than our baseline DSGE-2v model. For example, when we allow regimes associated with the variances of the two technology shocks to be independent of the regime-switching processes of the other shock variances (DSGE-2v2v), we obtain a Schwarz criterion of 5958.18, which is lower than that of the baseline DSGE-2v model (5963.03). In short, the data favor the parsimoniously parameterized model with shock variances switching regimes simultaneously.

TABLE 2. Comprehensive measures of model fits.

Model	Marginal Data Density
DSGE-con	5741.24
DSGE-2v	5832.38
DSGE-2c	5739.32
DSGE-2cv	5832.60
DSGE-2c2v	5830.84
DSGE-2v2v	5826.95
DSGE-3v	5813.91

The last column in Table 1 shows that the model with regime changes in shock variances only continues to dominate all the other models, when the persistence parameters in both price- and wage-markup shock processes are restricted to zero. In particular, the model with the target switching regimes (DSGE-2c) does not improve on the constant-parameter model. Of course, all these restricted models fit to the data much worse than the corresponding baseline models, implying that persistent shock processes are important in fitting the data.

Finally, we have estimated a number of models with persistence parameters in other shock processes set to zero and with habit and indexation parameters set to zero. The model with synchronized regimes in shock variances continues to outperform other models in fitting the data.

The relative performance of the alternative DSGE models in fitting the data does not change when we look at the marginal data density (MDD). Table 2 reports the MDD for each of the alternative models. The table shows that the model with simultaneous regime shifts in shock variances (DSGE-2v) is the best-fit model not only in terms of the Schwarz criterion, but also in terms of the marginal data density. In particular, the DSGE-2v model's MDD is 5832.38, which is much higher than that of the DSGE-con model (whose MDD is 5741.24). The model with regime switching in the inflation target alone (DSGE-2c) slightly outperforms the constant-parameter model, but substantially underperforms the DSGE-2v model. With regime shifts in shock variances, introducing regime shifts in the inflation target synchronized with regime shifts in shock variances (DSGE-2cv) or allowing the inflation target to follow a Markov switching process independent of shock regimes (DSGE-2c2v) does not improve the marginal data density relative to the DSGE-2v model.

Our estimation results attribute the goodness of fit for both DSGE-2cv and DSGE-2c2v models to significant shifts in shock variances, not shifts in inflation targets. In DSGE-2cv, the point estimates indicate that the inflation target is 2.18% for one regime and 1.70% for the other regime. In DSGE-2c2v, the inflation target is 2.4% for one regime and 2.1% for the other regime. The differences between the two inflation-target regimes are statistically insignificant. These results explain why incorporating regime shifts in

the inflation target in the presence of heteroskedasticity does not help improve the model's fit.[9]

6.2 *Estimates of structural parameters*

We first discuss our best-fit model DSGE-2v. The model is similar to that in Smets and Wouters (2007) with six notable exceptions. First, we introduce a source of real rigidity in the form of firm-specific factors; this replaces the kinked demand curves considered by Smets and Wouters (2007). Second, we introduce trend growth in the investment-specific technological change to better capture the data, in which the relative price of investment goods (e.g., equipment and software) has been declining for most of the postwar period, while in Smets and Wouters (2007), the investment-specific technological changes have no trend component. We use the observed time series of biased technological changes in our estimation, while Smets and Wouters (2007) treated these changes as a latent variable. Third, we introduce the depreciation shock that acts as a wedge in the capital-accumulation Euler equation. Fourth, the preference shock in our model enters all intertemporal decisions, including choices of the nominal bond, the capital stock, and investment, while Smets and Wouters (2007) introduced a "risk-premium shock" that enters the bond Euler equation only and does not affect other intertemporal decisions. Fifth, in the interest-rate rule, we assume that the nominal interest rate responds to deviations of inflation from its target and detrended output, while in Smets and Wouters (2007) the interest-rate rule targets inflation, output gap, and the growth rate of output gap. Finally, we allow for heteroskedasticity of structural shocks to obtain the accurate estimate of the role of a particular shock in explaining macroeconomic fluctuations. All these distinctions may explain some of the differences between our estimated results and theirs.

Tables 3 and 4 report the estimates of the model parameters. The data are informative about many structural parameters. Among the three preference parameters, the estimate for habit persistence (b) is 0.91 with the tight error bands. The estimate for η is 2.89, implying a Frisch elasticity of 0.35 and consistent with most microeconomic studies. The probability interval indicates that η can be as high as 8.38. The estimate for the subjective discount factor β is 0.998 (the same as the value obtained by Smets and Wouters (2007)) with the tight probability interval [0.996, 0.999].

Among the technology parameters, the estimate for α_1 (0.153) with the upper error band (0.216) is close to the estimate obtained by Smets and Wouters (2007) (0.19). Because of the constraint $\alpha_1 + \alpha_2 \leq 1$, the estimate for α_2 is 0.835. These posterior estimates suggest that the data prefer a model specification with (near) constant-returns production technology. The estimated trend growth rate for the investment-specific technological change (λ_q) is 4% per annum, slightly higher than the calibrated value obtained by Greenwood, Hercowitz, and Krusell (1997) because we include the data from the late 1990s until 2007 when the investment-specific technological improvement was the

[9]The posterior distribution is very non-Gaussian and full of local peaks. At some local peaks (whose posterior kernel values are more than 50 in log value below the posterior value), we obtain distinctly different target values.

TABLE 3. Prior and posterior distributions of structural parameters for the DSGE-2v and DSGE-2c Models.[a]

	Prior			Posterior (DSGE-2v)			Posterior (DSGE-2c)
Parameter	Distribution	5%	95%	Mode	5%	95%	Mode
b	Beta	0.05	0.948	0.907	0.8898	0.9533	0.877
α_1	Beta	0.15	0.35	0.163	0.1701	0.2162	0.153
α_2	Beta	0.35	0.75	0.835	0.7348	0.8101	0.837
η	Gamma	0.2	10.0	2.888	2.7151	8.3848	2.235
$100(\lambda_q - 1)$	Gamma	0.1	1.5	1.000	0.7663	1.5001	1.000
$100(\lambda_* - 1)$	Gamma	0.1	1.5	0.237	0.0876	0.4414	0.180
$100(\beta^{-1} - 1)$	Beta	0.2	4.0	0.175	0.0518	0.3915	0.109
σ_u	Gamma	0.5	3.0	2.263	1.3343	5.5196	2.266
S''	Gamma	0.5	5.0	2.000	1.2086	3.6145	1.525
$\mu_p - 1$	Gamma	0.01	0.50	0.000	0.0001	0.0032	0.002
$\mu_w - 1$	Gamma	0.01	0.50	0.060	0.0166	0.4151	0.069
4δ	Beta	0.05	0.2	0.134	0.0956	0.5928	0.107
ξ_p	Beta	0.1	0.75	0.412	0.2312	0.6202	0.545
γ_p	Beta	0.05	0.95	0.178	0.0261	0.3390	0.118
ξ_w	Beta	0.1	0.75	0.213	0.2482	0.6779	0.185
γ_w	Beta	0.05	0.95	1.000	0.2275	0.9724	1.000
ρ_r	Beta	0.05	0.948	0.816	0.7923	0.8726	0.730
ϕ_π	Gamma	0.5	5.0	1.655	1.4119	2.2899	1.186
ϕ_y	Gamma	0.05	3.0	0.043	0.0241	0.1168	0.006
$400 \log \pi^*(1)$	Gamma	1.0	8.0	2.283	1.2228	6.1118	4.981
$400 \log \pi^*(2)$	Gamma	1.0	8.0				1.982

[a]The column headings 5% and 95% demarcate the bounds of the 90% probability interval. DSGE-2v denotes the model with two regimes in shock variances and DSGE-2c denotes the model with two regimes in the inflation target.

fastest in the sample. The estimate for the trend growth rate of the neutral technological change (λ_*) is 0.95% per annum. There is a large amount of uncertainty about these trend estimates as shown in the last two columns of Table 3. The curvature parameter in the utilization function (σ_u) is estimated at 2.26, which is substantially lower than the value obtained by Justiniano and Primiceri (2008) (7.13), but higher than the values estimated by Altig et al. (2011) (2.02) and by Smets and Wouters (2007) (1.174). The error bands show a large amount of uncertainty around the estimate of this parameter. The investment adjustment cost parameter (S'') is estimated to be 2.0, which is lower than those obtained in the literature. Unlike most studies in the literature that fix the value of the capital depreciation rate a priori, we allow the depreciation rate δ to follow a stationary stochastic process and estimate the parameter in the process. The estimated average annum depreciation rate is 13.4%, which is remarkably close to the standard calibration value in the real business-cycle literature, but the error bands are very wide, implying the great uncertainty about this estimate.

Among the pricing and wage-setting parameters, the estimated average price markup (μ_p) is about 1.0, which is consistent with the studies by Hall (1988), Basu and Fernald (1997), and Rotemberg and Woodford (1999), who argued that the pure economic profit is close to zero. It is also similar to the estimate obtained by Altig et al.

TABLE 4. Prior and posterior distributions of shock process parameters for the DSGE-2v and DSGE-2c models.[a]

	Prior			Posterior (DSGE-2v)			Posterior (DSGE-2c)
Parameter	Distribution	5%	95%	Mode	5%	95%	Mode
ρ_p	Beta	0.05	0.948	0.949	0.8524	0.9642	0.998
ϕ_p	Beta	0.05	0.948	0.698	0.5009	0.8439	0.676
ρ_w	Beta	0.05	0.948	0.999	0.8047	0.9973	0.998
ϕ_w	Beta	0.05	0.948	0.749	0.6597	0.9307	0.645
ρ_{gz}	Gamma	0.2	3.0	0.894	0.4555	1.3748	1.446
ρ_a	Beta	0.05	0.948	0.107	0.0471	0.4010	0.061
ρ_q	Beta	0.05	0.95	0.994	0.9875	0.9973	0.994
ρ_z	Beta	0.05	0.95	0.992	0.9832	0.9984	0.997
ρ_d	Beta	0.05	0.948	0.934	0.9172	0.9820	0.899
$\sigma_r(1)$	Inverse gamma	0.0005	1.0	0.004	0.0038	0.0066	0.003
$\sigma_r(2)$	Inverse gamma	0.0005	1.0	0.001	0.0012	0.0016	
$\sigma_p(1)$	Inverse gamma	0.0005	1.0	0.039	0.0312	0.1426	0.024
$\sigma_p(2)$	Inverse gamma	0.0005	1.0	0.028	0.0211	0.0819	
$\sigma_w(1)$	Inverse gamma	0.0005	1.0	0.255	0.2708	2.9007	0.106
$\sigma_w(2)$	Inverse gamma	0.0005	1.0	0.144	0.1452	1.6253	
$\sigma_g(1)$	Inverse gamma	0.0005	1.0	0.041	0.0332	0.0566	0.026
$\sigma_g(2)$	Inverse gamma	0.0005	1.0	0.021	0.0192	0.0247	
$\sigma_z(1)$	Inverse gamma	0.0005	1.0	0.010	0.0099	0.0167	0.008
$\sigma_z(2)$	Inverse gamma	0.0005	1.0	0.006	0.0064	0.0083	
$\sigma_a(1)$	Inverse gamma	0.0005	1.0	0.043	0.0375	0.1371	0.028
$\sigma_a(2)$	Inverse gamma	0.0005	1.0	0.037	0.0337	0.0819	
$\sigma_q(1)$	Inverse gamma	0.0005	1.0	0.007	0.0064	0.0109	0.004
$\sigma_q(2)$	Inverse gamma	0.0005	1.0	0.002	0.0026	0.0034	
$\sigma_d(1)$	Inverse gamma	0.0005	1.0	0.193	0.1442	1.0925	0.139
$\sigma_d(2)$	Inverse gamma	0.0005	1.0	0.099	0.0702	0.4989	
q_{11}	Dirichlet	0.589	0.991	0.807	0.6045	0.8749	0.810
q_{22}	Dirichlet	0.589	0.991	0.940	0.9063	0.9769	0.993

[a]The column headings 5% and 95% demarcate the bounds of the 90% probability interval. DSGE-2v denotes the model with two regimes in shock variances and DSGE-2c denotes the model with two regimes in the inflation target.

(2011), but much smaller than the value estimated by Justiniano and Primiceri (2008). Our estimate for the average wage markup (μ_w) is 1.06, which is lower than the calibrated value (Huang and Liu (2002)) and the estimated value (Justiniano and Primiceri (2008)), but is similar to the value used by Christiano, Eichenbaum, and Evans (2005). The uncertainty about the wage-markup parameter, judged by the 0.90 probability bands, is much larger than the uncertainty about the price-markup parameter. The estimated price- and wage-stickiness parameters ($\xi_p = 0.412$ and $\xi_w = 0.213$) imply that, on average, price contracts last for less than two quarters and nominal wage contracts have an even shorter duration, which is slightly more than one quarter. Our estimated nominal contract duration is consistent with microeconomic studies such as Bils and Klenow (2004). The estimated dynamic indexation is unimportant for price setting ($\gamma_p = 0.178$), but very important for nominal wage setting ($\gamma_w = 1.0$). The 0.90 probability intervals

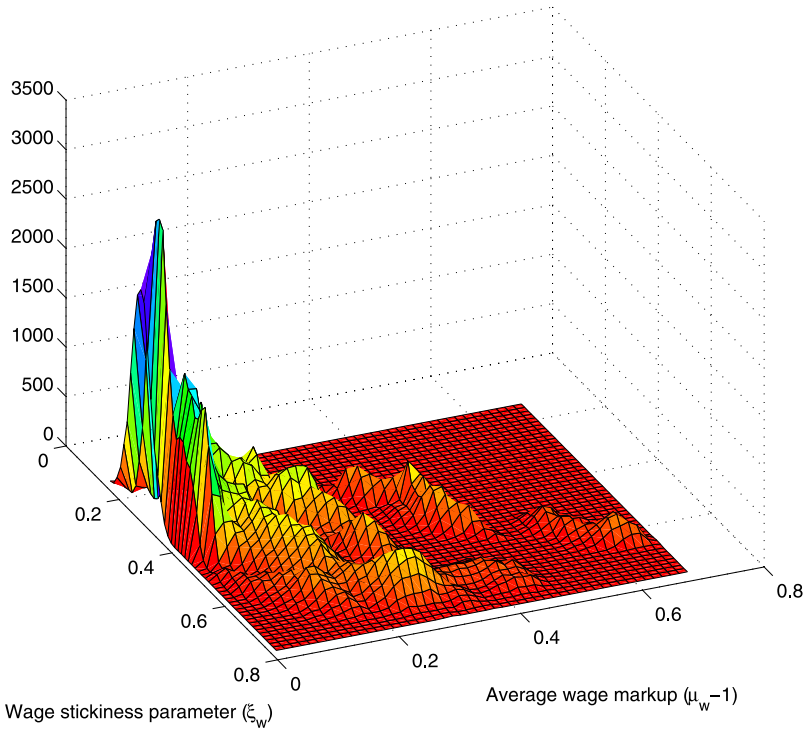

FIGURE 1. The histogram plot for the joint posterior distribution of the wage-stickiness parameter and the average wage-markup parameter for the model DSGE-2v.

indicate that while the price indexation is tightly estimated, the uncertainty about the nominal wage indexation is extremely large.

As shown in Table 3, the estimated wage-stickiness parameter lies below the lower bound of the 0.90 probability interval. This phenomenon occurs because the posterior distribution around the mode for this parameter is on the thin ridge and because there are many local peaks that give a significant probability to regions containing the values above the estimated wage-stickiness parameter. While it is impossible to graph this phenomenon in a high dimensional parameter space like ours, we display in Figure 1 the joint distribution of the wage-stickiness parameter and the average wage-markup parameter after integrating out all other parameters. As can be seen, the multiple local peaks give much of the probability to the values of the wage-stickiness parameters greater than the estimate at the posterior mode. Because the two-dimensional distribution displayed in Figure 1 integrates out all other parameters, the distribution is already skewed toward the values of the wage-stickiness parameters greater than 0.2. Nonetheless, the picture demonstrates clearly the nature of thin ridges and multiple local peaks inherent in the posterior distribution.

The estimates of policy parameters suggest that interest-rate smoothing is important; the estimate of ρ_r is 0.82 with a narrow probability interval. The policy response to deviations of inflation from its target in the interest rule (ϕ_π) is 1.655 with the lower probability bound still significantly above 1.0. Policy does not respond much to detrended output and the parameter (ϕ_y) is tightly estimated. The inflation target (π^*) is estimated at 2.28% per annum.

The estimated results for shock processes are reported in Table 4. The AR(1) coefficients for all shocks except the preference shock (ρ_a) are above 0.9, although the lower probability bounds for some coefficients are substantially below (0.9). The preference shock is almost i.i.d. The MA(1) coefficients in the price-markup and wage-markup processes (ϕ_p and ϕ_w) are both sizable. The estimates are 0.698 and 0.749, and the corresponding 0.90 probability intervals support these high values. The government spending shock responds to the neutral technology shock; the response coefficient (ρ_{gz}) is 0.894 with a wide probability interval. Although the prior distributions for all the shock variances are the same, the posterior estimates are very disperse. The depreciation shock (σ_d) and the wage-markup shock (σ_w) have the largest variances; the monetary policy shock (σ_r) and the two types of technology shocks (σ_z and σ_q) have the smallest variances. The 0.90 probability intervals indicate that the marginal posterior distribution of a shock variance is skewed to the right. This shape is expected as the variance is bounded below by zero and has no upward bound.

As shown in Table 4, the estimated shock variances in the second regime are substantially smaller than those in the first regime. The estimated transition probabilities are summarized by the matrix

$$\hat{Q} = \begin{bmatrix} 0.8072 & 0.0598 \\ 0.1928 & 0.9402 \end{bmatrix}, \tag{49}$$

where the elements in each column sum to 1. The second regime (i.e., the regime with low shock variances) is more persistent and, as shown in Figure 2, covers most of the

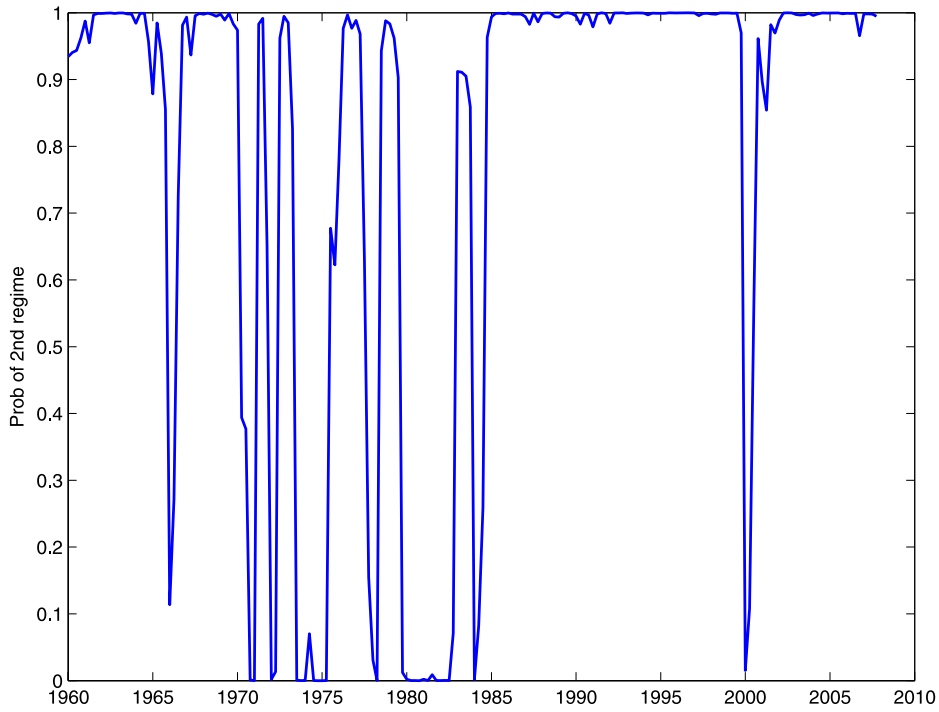

FIGURE 2. Posterior probabilities of the less volatile regime (the second regime) for the DSGE-2v model.

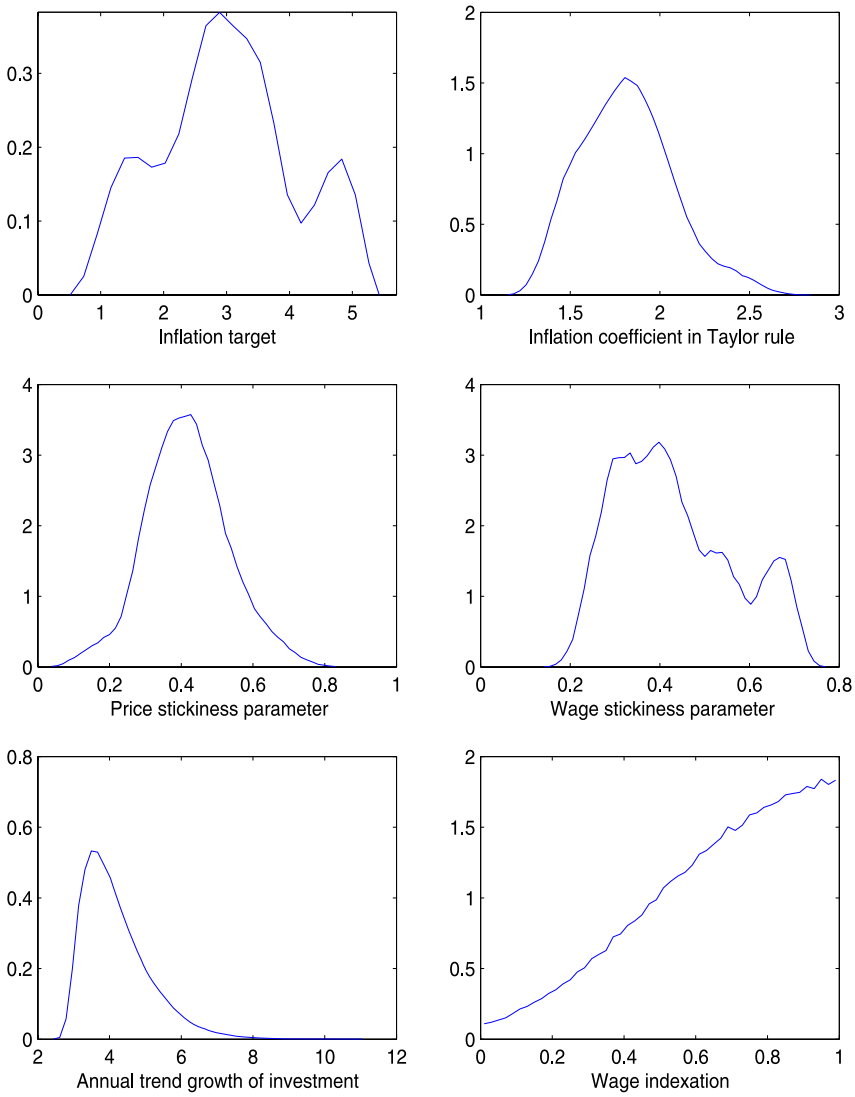

FIGURE 3. Marginal posterior distributions of some key parameters for the DSGE-2v model.

period since Greenspan became Chairman of the Federal Reserve Board. This result is even stronger when we take into account the error bands, where the lower bound of q_{22} is higher than the upper bound of q_{11}.

Figure 3 plots the marginal posterior distribution of some key parameters. The local peaks shown in the marginal distribution of the inflation target are the direct outcome of the integrated effect of the non-Gaussian joint posterior distribution of all parameters that have thin ridges and multiple peaks. Most of the probability, however, concentrates between 2% and 4%. The marginal distribution of the response coefficient to inflation in the Taylor rule indicates that there is practically no probability for indeterminate equilibria for our model.

The marginal distribution of the price-stickiness parameter implies that the price rigidity is much smaller than that obtained in the previous literature. The posterior mode is near the lower tail of the marginal distribution. The joint distribution, as il-

lustrated in Figure 1, has a thin ridge and many local peaks. After integrating out all other parameters, the marginal distribution of the wage-stickiness parameter shows a local peak around 0.7. The majority of the probability, however, lies below the value 0.6. There are two reasons why we obtain estimates that imply shorter durations of price and wage contracts than those obtained in the literature such as Smets and Wouters (2007) and Altig et al. (2011). First, our estimates suggest that the price markup is very small, implying that the demand curve for differentiated goods is very flat. Thus, a small increase in the relative price can lead to large declines in relative output demand. Even if firms can reoptimize their pricing decisions very frequently, they choose not to adjust their relative prices too much. In this sense, the small average markup and thus the large demand elasticity become a source of strategic complementarity in firms' pricing decisions. Second, unlike Justiniano and Primiceri (2008), who used a minimum-distance estimator that matches the model's impulse responses to those in the data, we use full-information maximum likelihood estimation. This difference is important because Justiniano and Primiceri (2008) found that, while a shock to neutral technology leads to rapid adjustments in prices, a shock to monetary policy leads to small and gradual price adjustments. Under their estimation approach, matching the impulse responses following the monetary policy shock is important so that price adjustments have to be small and gradual. Our estimation approach differs from theirs and we find that the most important shocks are those to neutral technology, capital depreciation, and wage markup, all of which lead to rapid adjustments in prices. Consequently, our estimated durations of nominal contracts are shorter than those in the literature.

The last row of Figure 3 displays the marginal posterior distributions of the investment technology trend and the wage indexation. The distribution of the investment technology trend puts a significant amount of probability around 4%, consistent with the data on the relative price of investment. The distribution of the wage indexation parameter is most interesting. While the estimate is at 1.0, there is considerable uncertainty around the wage indexation parameter so that the estimate of 1.0 is very imprecise. This result implies that our estimation does not necessarily support a strong wage indexation.

7. Regime switches in the inflation target

In this section, we discuss the key results from the DSGE model with two regimes in the inflation target. There are several reasons to study the implications from this model, even though its fit is dominated by the DSGE model with regime changes in shock variances. First, this type of regime-switching model is in line with the previous studies on changes in the inflation target (Erceg and Levin (2003), Ireland (2005), Schorfheide (2005)). Second, the model seems most relevant to analyzing the effect of a change in the inflation target, as it gives us the best chance to detect changes in the target regime. Third, it is informative to determine, in light of the model's results, the extent to which changes in the inflation target explain the rise and fall of high inflation in the 1970s.

7.1 *The baseline model*

As discussed in Section 6.1, when we fit the models DSGE-2cv and DSGE-2c2v to the data, the data attribute most of regime changes to shifts in shock variances and little to different inflation-target values. The finding, however, is different when we fit DSGE-2c to the data.

The last columns in Tables 3 and 4 report the parameter estimates for the baseline DSGE-2c model. Most estimated structural parameters are similar to those in the DSGE-2v model, although the estimated Calvo price-stickiness parameter is higher, and the estimated interest-rate smoothing parameter and the inflation coefficient in the Taylor rule are lower than those in the DSGE-2v model. The estimated shock standard deviations in the DSGE-2c model lie mostly between the standard deviations in the two shock regimes, while the standard deviations of the price-markup shock, the wage-markup shock, and the preference shock in the DSGE-2c model are lower than those in the low-variance regime in the DSGE-2v model.

The estimates in the DSGE-2c model give two distinct inflation-target regimes, with a high target of about 5% annualized inflation and a low target of about 2%. The question is to what extent such regime changes in the inflation target help explain high inflation in the 1970s. To answer this question, we conduct two exercises.

First, we compute the posterior probability of the high-target regime throughout history. As shown in the upper panel of Figure 4, the probability of the high-target regime is near 1 in the first wave of great inflation in the early 1970s, while the probability is near 0 in other periods. Sargent, Williams, and Zha (2006) and Carboni and Ellison (2009) argued that government's misconception about the trade-off between unemployment and inflation in the early 1970s was the most important driving force of high inflation. Although all agents in our model, unlike Sargent, Williams, and Zha (2006) and Carboni and Ellison (2009), are fully rational, the finding of the high-inflation target in the early 1970s corroborates their view.

Second, we compute counterfactual paths of inflation by fixing the inflation target at either the low-target regime or the high-target regime throughout the sample.[10] In our computation, we use the estimated DSGE-2c model to back out the posterior historical shocks and the posterior probabilities of each inflation-target regime throughout the sample. Thus, by construction, had we weighed inflation-target regimes by their posterior probabilities at each time t and left the posterior historical shocks in place, the simulated inflation path would have exactly matched the actual data.[11] In our counterfactual exercises where we fix the inflation target to a particular level, the simulated inflation path would, in general, be different from the actual data.

These counterfactual results show that the existence of the high-target regime and the probability of switching to that regime are important for explaining the first wave

[10] Since the ergodic probability of the low-target regime is over 95%, the unconditional mean of the inflation target is very close to the low-inflation target.

[11] This result is true only for DSGE models with regime switches in shock variances or in constant terms such as the inflation target. It does not hold true, in general, if some coefficients in an equation are switching regimes.

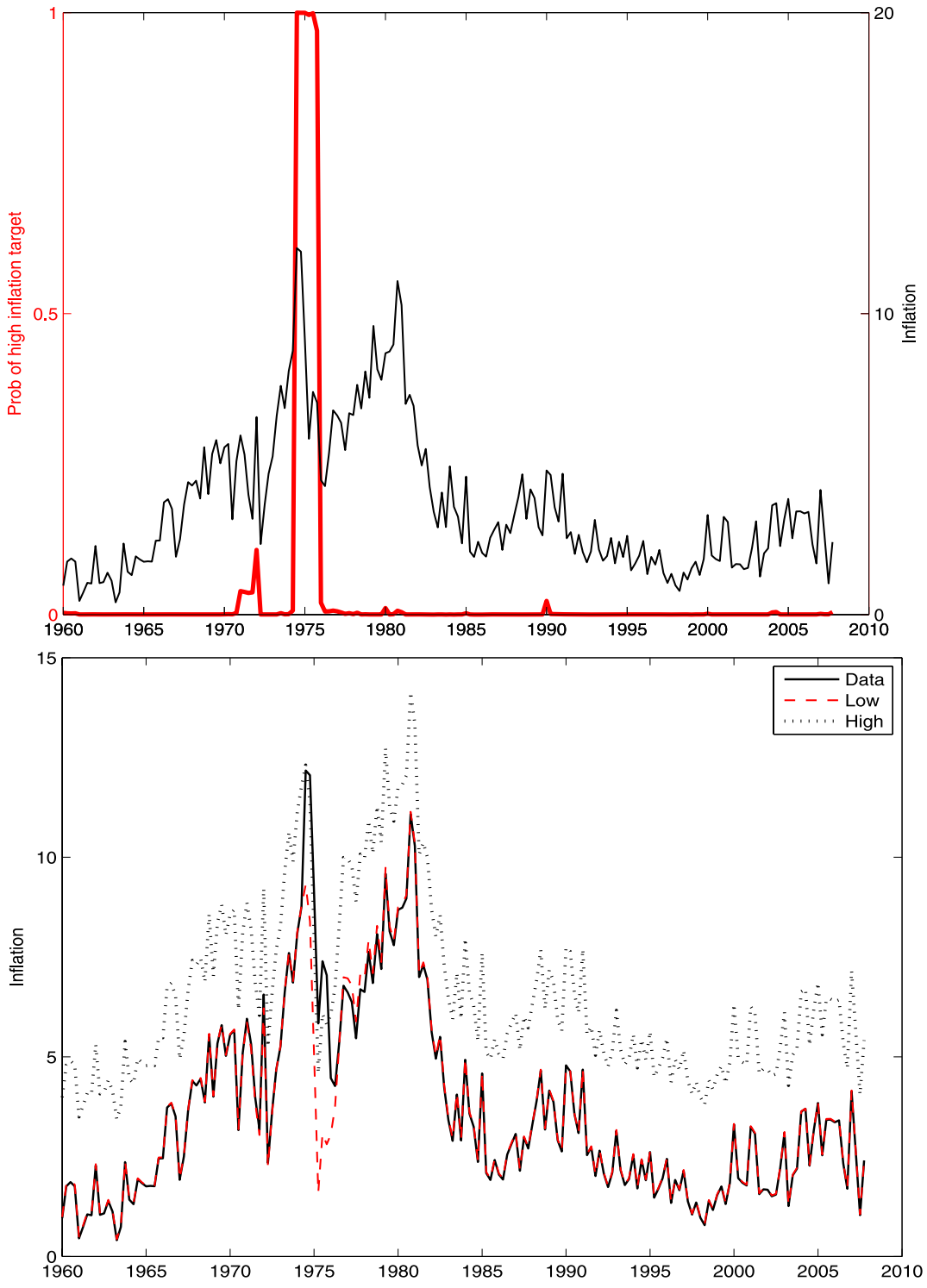

FIGURE 4. Top panel: Posterior probabilities of the high-inflation-target regime for the DSGE-2c model (on the left scale) and actual inflation data (on the right scale). Bottom panel: Counterfactual paths of inflation simulated from the DSGE-2c model when the inflation target was fixed at the low-target regime (the dashed line) and at the high-target regime (the dotted line) throughout the sample.

of high inflation in the 1970s, consistent with the findings in Erceg and Levin (2003), Ireland (2005), Schorfheide (2005), Sargent, Williams, and Zha (2006), and Carboni and Ellison (2009). The lower panel of Figure 4 shows that had the low-target regime prevailed throughout history, the peak inflation rate would have been less than 9% in 1974, much lower than the actual inflation rate of about 12%, and inflation would have dropped to 2% in 1975 and stayed low through 1976 (compare the solid and dashed lines in the figure).

For the late 1970s and early 1980s, however, the counterfactual inflation path under the permanent low-target regime tracks the actual path almost exactly. Thus, the model does not attribute the run up of inflation in the late 1970s to a high-inflation target. This result is consistent with the findings of Sims and Zha (2006), who conducted a similar exercise but in the context of structural Bayesian VAR (BVAR) models instead of DSGE models.

These results, however, by no means imply that changes in the inflation target do not matter in some abstract sense. As shown in the lower panel of Figure 4, had the inflation target been fixed at the high-target regime throughout history, the counterfactual path of inflation (the dotted line) would have been substantially higher than the actual path throughout the sample. What is critical is that the estimated duration of high-inflation target is very short. The estimated probabilities of the two inflation-target regimes (the bottom two numbers in the last column of Table 4) imply that the ergodic (unconditional) probability of the high-target regime is much smaller than that of the low-target regime (3.55% vs. 96.45%).

In summary, a richly parameterized DSGE model, coupled with a large number of shock processes like our DSGE-2c model, is likely to attribute the rise and fall of inflation more to the shock processes than to changes in monetary policy (in our case, regime changes in the inflation target). This point is not new; it already was made by Sims and Zha (2006). Our counterfactual exercises simply reinforce this point in the context of relatively large DSGE models.

7.2 *Alternative specification*

In our model with regime changes in the inflation target, we take the standard forms for the price and wage indexation rules as well as for the interest-rate rule. Specifically, in (7) and (20), the price and the nominal wage are each indexed partially to the steady-state inflation (i.e., the unconditional mean of inflation), and in (25), the intercept of the interest-rate rule concerns the steady-state inflation.

It is useful to consider an alternative specification in which the price and wage indexation rules and the interest-rate rule reflect regime changes in the inflation target. The central idea is that this alternative specification potentially allows additional persistence to the inflation process and thus improves the fit of the model that allows for regime changes in the inflation target (i.e., the DSGE-2c model studied in the previous

section). The alternative specifications are

$$P_t(j) = \pi_{t-1}^{\gamma_p} \pi^*(s_t)^{1-\gamma_p} P_{t-1}(j), \tag{50}$$

$$W_t(h) = \pi_{t-1}^{\gamma_w} \pi^*(s_t)^{1-\gamma_w} \lambda_{t-1,t}^* W_{t-1}(h), \tag{51}$$

$$R_t = \kappa R_{t-1}^{\rho_r} \left[r\pi^*(s_t) \left(\frac{\pi_t}{\pi^*(s_t)} \right)^{\phi_\pi} \left(\frac{Y_t}{\lambda_t^*} \right)^{\phi_y} \right]^{1-\rho_r} e^{\sigma_{rt}\varepsilon_{rt}}, \tag{52}$$

where κ is the same as in (25). In comparison to (7), (20), and (25), the steady-state inflation rate π is now replaced by the regime-switching inflation target $\pi^*(s_t)$.

Incorporating these changes leads to changes in the log-linearized equations of the price–Phillips curve (31), the wage–Phillips curve (33), and the Taylor rule (43). These equations are now replaced by

$$\begin{aligned} &\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} - (1-\gamma_p)\hat{\pi}_t^* \\ &\quad = \frac{\kappa_p}{1+\bar{\alpha}\theta_p} (\hat{\mu}_{pt} + \hat{\mathrm{mc}}_t) + \beta \mathrm{E}_t [\hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t - (1-\gamma_p)\hat{\pi}_{t+1}^*], \end{aligned} \tag{53}$$

$$\begin{aligned} &\hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t - \gamma_w \hat{\pi}_{t-1} - (1-\gamma_w)\hat{\pi}_t^* \\ &\quad = \frac{\kappa_w}{1+\eta\theta_w} (\hat{\mu}_{wt} + \hat{\mathrm{mrs}}_t - \hat{w}_t) \\ &\qquad + \beta \mathrm{E}_t [\hat{w}_{t+1} - \hat{w}_t + \hat{\pi}_{t+1} - \gamma_w \hat{\pi}_t - (1-\gamma_w)\hat{\pi}_{t+1}^*], \end{aligned} \tag{54}$$

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1-\rho_r)[\phi_\pi \hat{\pi}_t + (1-\phi_\pi)\hat{\pi}_t^* + \phi_y \hat{y}_t] + \sigma_{rt}\varepsilon_{rt}, \tag{55}$$

where $\hat{\pi}_t^*$ denotes the deviations of the inflation target from its ergodic mean.[12]

The alternative specification improves the fit of all three models DSGE-2cv, DSGE-2c2v, and DSGE-2c relative to the baseline specification (in the range of 10–15 in log value by both the Schwarz criterion and the marginal data density). These results indicate that the alternative specification is not only sensible, but also favored by the data.

The posterior estimates for most parameters under the alternative specification are very similar to those under the baseline specification. When we allow regime shifts in both the inflation target and the shock variances (DSGE-2cv and DSGE-2c2v), we obtain estimates of the two inflation targets under the alternative specification that are nearly identical to those under the baseline specification. Although estimates of the transition probabilities of inflation-target regimes in the DSGE-2c2v model are different from the baseline specification, inflation dynamics are driven by regime changes in volatility, not by changes in the inflation target.

When we keep shock variances unchanged while allowing the inflation target to switch regimes (DSGE-2c), the difference between two estimated inflation targets is large enough for changes in the transition matrix to matter in practice. The estimation results show that $q_{11} = 0.919$ and $q_{22} = 0.992$ under the alternative dynamic indexation and the Taylor rule (compared to $q_{11} = 0.810$ and $q_{22} = 0.993$ in the baseline model).

[12]See the online appendix for details of the derivation, available in a supplementary file on the journal website at http://qeconomics.org/supp/71/supplement.pdf.

The implied ergodic probability for the high-target regime increases from 3.55% in the baseline model to 8.99% in the alternative model. As in the baseline DSGE-2c model, the probability of the high-target regime is near 1 in the first wave of great inflation in the early 1970s. Unlike the baseline model, the probability of the high-target regime also rises briefly to about 0.5 in the beginning of the 1980s and again to about 0.2 in the period from 2003 to 2006 (see the upper panel of Figure 5).

Although the high-target regime in the alternative model appears to occur more frequently than in the baseline model, our counterfactual simulation suggests that the existence of the high-target regime becomes less important for explaining the actual path of inflation than under our baseline specification. In particular, the lower panel of Figure 5 shows that had the low-target regime prevailed throughout the sample periods, the inflation rate would not have been much different from actual history (except for the small dips in the mid 1970s). This result is somewhat surprising, since the dynamic indexation of price- and wage-setting rules and the interest-rate feedback rule all respond to changes in the inflation-target regime. On the other hand, had the high-target regime prevailed throughout history, inflation would have been much higher than the actual data, especially in the mid 1970s (compare the dotted line with the solid line in the lower panel of Figure 5); moreover, the counterfactual inflation path is much higher under this alternative specification than under the baseline specification (compare the dotted lines in the lower panels of Figures 4 and 5). Overall, our counterfactual exercises with this alternative specification confirm the main finding that in a rich DSGE model like ours, the shock processes are more likely to be a main driving force of the rise and fall in inflation than are changes in the inflation target.

8. Economic implications

We now discuss the economic implications of our best-fit model. We first examine, in Section 8.1, the role of the various shocks in driving macroeconomic fluctuations through variance decompositions. We then present, in Section 8.2, impulse responses of several key aggregate variables to each of the shocks that we identify as important for macroeconomic fluctuations. Finally, we provide some economic interpretations of the key sources of shocks and in particular, the capital depreciation shock.

8.1 *Variance decompositions*

Tables 5 and 6 report variance decompositions in forecast errors of output, investment, hours, the real wage, and inflation under the two shock regimes at different forecasting horizons for our best-fit model. As we discussed in Section 6.2, the wage-markup shock and the depreciation shock have the largest variances among all eight structural shocks. The neutral technology shock is of considerable interest because of the debate in the recent literature about its dynamic effects on the labor market variables (e.g., Galí (1999), Christiano, Eichenbaum, and Vigfusson (2003), Uhlig (2004), and Liu and Phaneuf (2007)).

As we can see, capital depreciation shocks, neutral technology shocks, and wage-markup shocks play an important role in driving business-cycle fluctuations under both

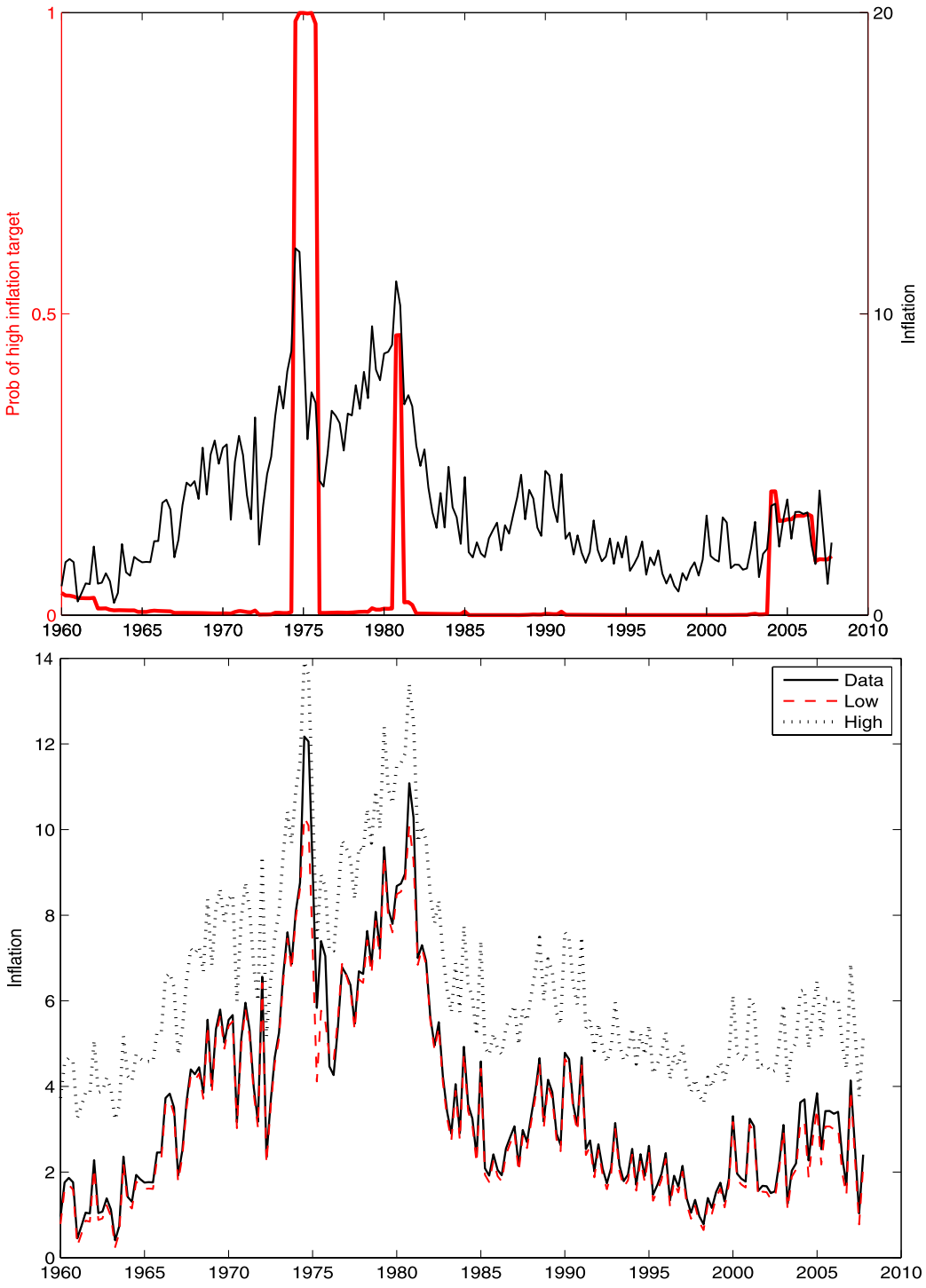

FIGURE 5. Top panel: Posterior probabilities of the high-inflation-target regime for the DSGE-2c model with the alternative specification (on the left scale) and actual inflation data (on the right scale). Bottom panel: Counterfactual paths of inflation simulated from the DSGE-2c model with the alternative specification when the inflation target was fixed at the low-target regime (the dashed line) and at the high-target regime (the dotted line) throughout the sample.

TABLE 5. Forecast error variance decomposition: Regime I.[a]

Horizon	MP	PM	WM	GS	Ntech	Pref	Btech	Dep
				Output				
4Q	5.1443	4.1486	21.7621	12.8076	21.7050	1.4694	0.2106	32.7525
8Q	2.6618	4.1472	36.6626	6.3157	23.4311	0.5821	0.3289	25.8707
16Q	1.2227	2.9278	49.3857	3.4905	25.7249	0.2658	0.3942	16.5885
20Q	0.9756	2.5166	52.4044	2.9881	26.1211	0.2121	0.4119	14.3702
				Investment				
4Q	8.2082	5.7979	14.4682	0.7438	4.7909	0.5182	1.5667	63.9061
8Q	4.6292	6.2905	22.8817	1.1375	7.1533	0.7034	2.2140	54.9904
16Q	3.2021	5.9545	30.5410	1.2310	10.2411	0.6839	3.6330	44.5134
20Q	3.0318	5.7789	32.0371	1.1764	11.1034	0.6478	4.2787	41.9460
				Hours				
4Q	6.2409	4.6560	33.5268	18.7046	3.9659	2.0046	0.0815	30.8198
8Q	3.4031	4.8188	58.8519	10.9127	1.8040	0.9368	0.1388	19.1339
16Q	1.7212	3.0812	76.6957	6.8522	0.9926	0.4698	0.1114	10.0761
20Q	1.3929	2.5430	79.8568	6.0511	0.8162	0.3841	0.0938	8.8619
				Real Wage				
4Q	6.2726	14.9794	24.5772	0.2213	34.1127	1.1717	0.2982	18.3668
8Q	5.6839	19.8200	12.8581	0.1161	31.7275	0.5643	0.2978	28.9321
16Q	3.3235	20.6873	6.9996	0.1603	36.8344	0.3290	0.3967	31.2691
20Q	2.8272	19.7771	5.9603	0.1726	39.2705	0.2857	0.4580	31.2486
				Inflation				
4Q	17.1586	11.2160	34.3253	1.1835	0.1755	1.1923	0.5994	34.1493
8Q	17.1334	9.3782	36.9222	1.1124	0.1509	1.0689	0.7772	33.4568
16Q	14.3407	7.9953	40.9412	0.9402	0.1589	0.9094	0.7557	33.9585
20Q	12.8011	7.2902	42.2714	0.8484	0.2053	0.8251	0.6802	35.0783

[a]The second through ninth columns correspond to the shocks: the monetary policy shock (MP), the price-markup shock (PM), the wage-markup shock (WM), the government spending shock (GS), the neutral technology shock (Ntech), the preference shock (Pref), the biased technology shock (Btech), and the depreciation shock (Dep).

regimes. Taken together, these three types of shocks account for 70%–80% of the fluctuations in output, investment, hours, and inflation under each regime for the forecast horizons beyond eight quarters. Monetary policy shock accounts for a sizable fraction of inflation fluctuations under the first regime, but otherwise it is unimportant. The price-markup shock contributes to about 15%–30% of the real wage fluctuations under both regimes. It is also somewhat important for inflation fluctuations under the second regime. The remaining three shocks, including the government spending shock, the preference shock, and the biased technology shock are unimportant in explaining macroeconomic fluctuations.

8.2 *Impulse responses*

To gain intuition about the model's transmission mechanisms, we analyze impulse responses of selected variables following some of the structural shocks. In particular, we focus on the dynamic effects of a wage-markup shock, a neutral technology shock, and

TABLE 6. Forecast error variance decomposition: Regime II.[a]

Horizon	MP	PM	WM	GS	Ntech	Pref	Btech	Dep
				Output				
4Q	1.4623	6.8572	22.2957	10.9572	27.1732	3.5101	0.0921	27.6522
8Q	0.7325	6.6368	36.3663	5.2312	28.4007	1.3462	0.1392	21.1470
16Q	0.3285	4.5746	47.8282	2.8228	30.4437	0.6002	0.1629	13.2390
20Q	0.2611	3.9167	50.5532	2.4070	30.7917	0.4770	0.1696	11.4238
				Investment				
4Q	2.6142	10.7376	16.6082	0.7130	6.7202	1.3870	0.7675	60.4524
8Q	1.3975	11.0425	24.8966	1.0335	9.5109	1.7845	1.0280	49.3065
16Q	0.9411	10.1760	32.3509	1.0888	13.2559	1.6891	1.6423	38.8559
20Q	0.8887	9.8499	33.8462	1.0379	14.3342	1.5957	1.9291	36.5185
				Hours				
4Q	1.8550	8.0476	35.9182	16.7333	5.1919	5.0074	0.0373	27.2093
8Q	0.9744	8.0232	60.7354	9.4043	2.2750	2.2543	0.0611	16.2724
16Q	0.4847	5.0454	77.8436	5.8075	1.2310	1.1118	0.0482	8.4277
20Q	0.3918	4.1596	80.9619	5.1229	1.0112	0.9081	0.0406	7.4039
				Real wage				
4Q	1.5771	21.9006	22.2722	0.1675	37.7754	2.4758	0.1153	13.7161
8Q	1.4263	28.9214	11.6296	0.0877	35.0659	1.1900	0.1150	21.5641
16Q	0.8150	29.4979	6.1863	0.1183	39.7808	0.6781	0.1496	22.7740
20Q	0.6917	28.1379	5.2561	0.1271	42.3184	0.5875	0.1724	22.7089
				Inflation				
4Q	5.3155	20.2044	38.3260	1.1035	0.2395	3.1041	0.2856	31.4214
8Q	5.3827	17.1324	41.8078	1.0518	0.2089	2.8220	0.3755	31.2189
16Q	4.4593	14.4569	45.8850	0.8799	0.2176	2.3764	0.3614	31.3634
20Q	3.9610	13.1172	47.1437	0.7901	0.2798	2.1456	0.3237	32.2388

[a]The column abbreviations are the same as in Table 5.

a depreciation shock on output, investment, the real wage, the inflation rate, hours, and the nominal interest rate. These shocks, as we discussed in the previous section, are the most important driving sources of macroeconomic fluctuations. Since the impulse responses display the same patterns for both shock regimes except the scaling effect, we report only the impulse responses for the second regime.

Figure 6 displays the impulse responses following a 1-standard-deviation shock to the capital depreciation rate. The increase in the depreciation rate reduces the value of capital accumulation, and raises utilization and the rental price of capital; thus investment falls. Since the expected stock of capital wealth declines, the negative wealth effect leads to a fall in consumption as well. Consequently, aggregate output falls. The decline in output leads to a decline in hours. The decline in hours and in consumption lowers the marginal rate of substitution between labor and consumption, so that the households' desired wage falls. Thus, the equilibrium real wage declines as well. The fall in the real wage reduces the firms' marginal cost so that inflation declines. Through the Taylor rule, the nominal interest rate declines as well. As the 0.90 probability error bands show, all the responses are statistically significant.

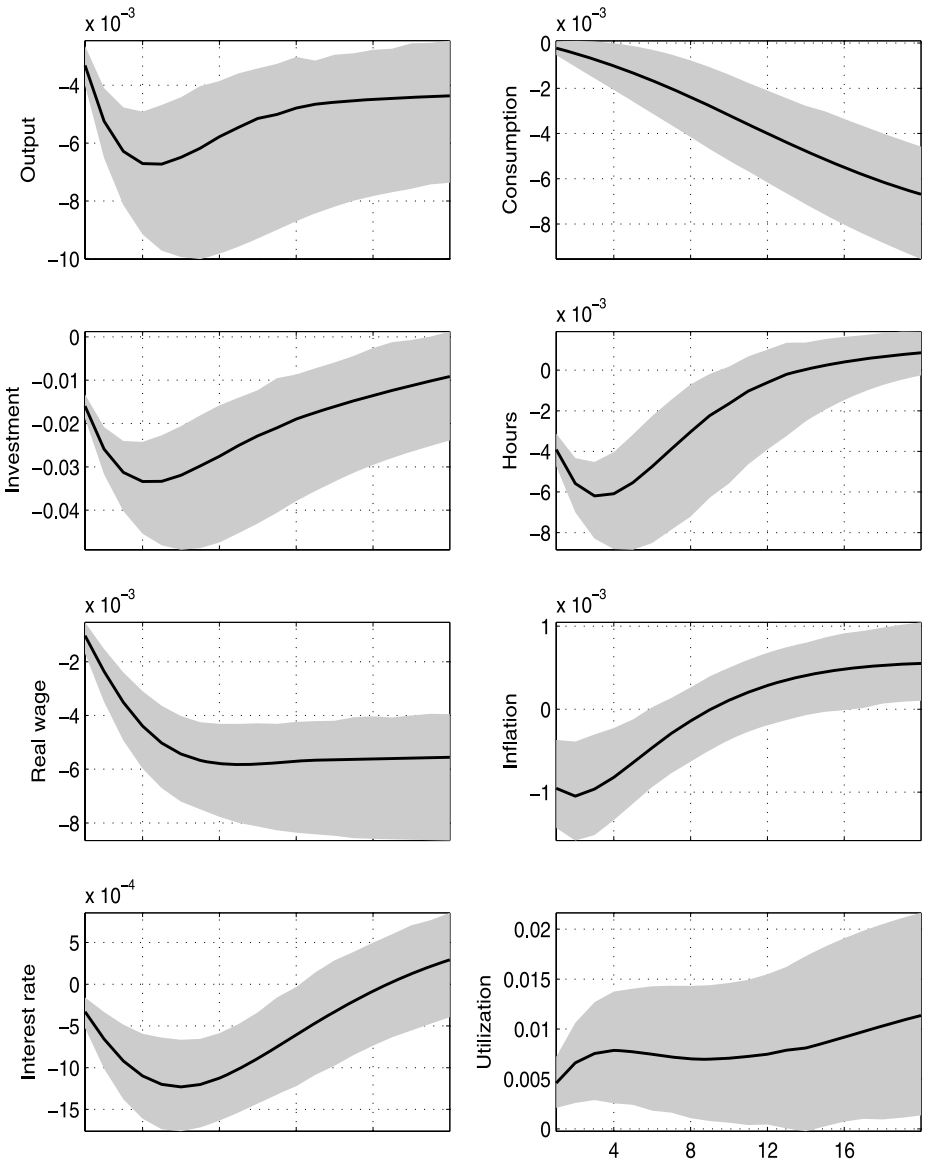

FIGURE 6. Impulse responses to a depreciation shock in the second regime for the DSGE-2v model. The shaded area represents 0.90 probability pointwise error bands and the thick line represents the median estimate.

Figure 7 reports the impulse responses following a 1-standard-deviation shock to the investment-specific technology. The biased shock raises the efficiency of investment, investment goods today become cheaper, and current consumption becomes more expensive. This type of shock, unlike the depreciation shock or the neutral technology shock, shifts resources from consumption to investment. Consequently, investment rises and consumption declines. Hours declines initially due to the costly adjustment in investment as well as the habit formation. After the second quarter, the increase in demand for investment gradually leads to a rise in hours and the real wage. The rise in labor hours helps produce more output. Utilization and the rental price of capital rise as well. All the responses are well estimated, judged by the 0.90 probability error bands. In con-

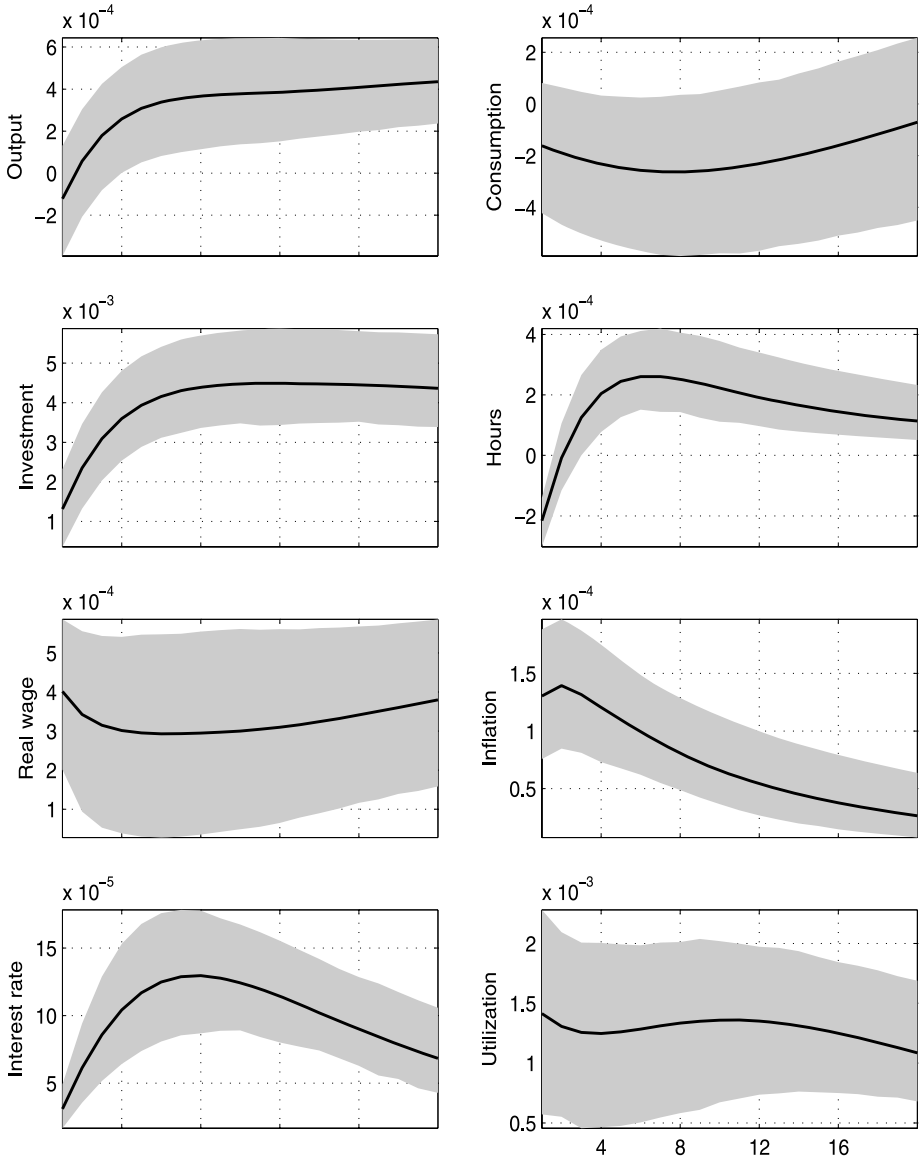

FIGURE 7. Impulse responses to an investment-specific technology shock in the second regime for the DSGE-2v model. The shaded area represents 0.90 probability pointwise error bands and the thick line represents the median estimate.

trast to the responses to the depreciation shock, the biased technology shock generates opposite movements in output and consumption in the short run; consequently, its impact on the macroeconomy is much smaller (by comparing the scales in Figures 6 and 7).

Both the capital depreciation shock and the investment-specific technology shock enter the intertemporal capital accumulation decision, but we find that this biased technology shock is much less important for macroeconomic fluctuations than the depreciation shock. This finding is different from that in Justiniano, Primiceri, and Tambalotti (2011), mainly because we use direct observations on the biased technology shock in our estimation, whereas they do not.

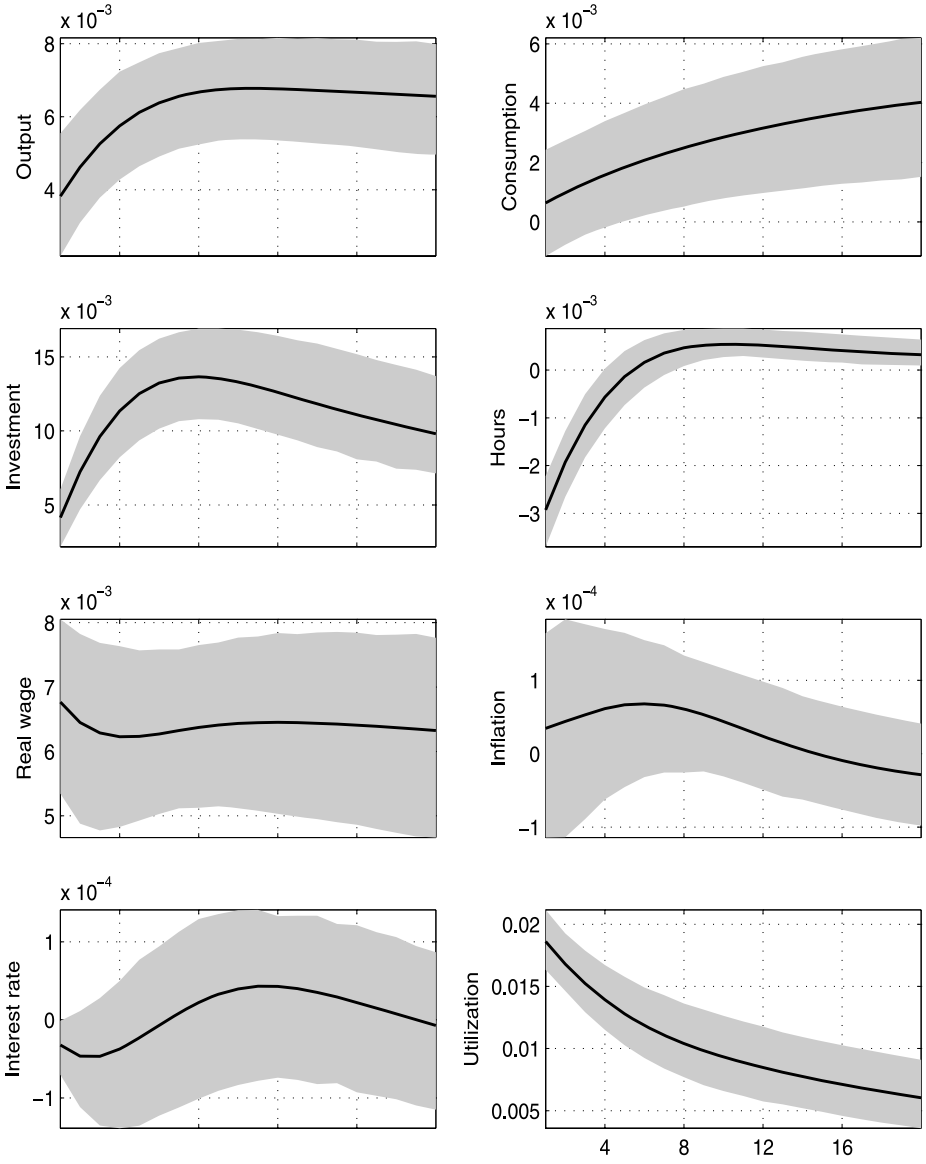

FIGURE 8. Impulse responses to a neutral technology shock in the second regime for the DSGE-2v model. The shaded area represents 0.90 probability pointwise error bands and the thick line represents the median estimate.

Figure 8 displays the impulse responses following a 1-standard-deviation shock to the neutral technology (i.e., the total factor productivity (TFP)). The positive neutral technology shock raises output, consumption, investment, utilization of capital, and the real wage. All these responses are statistically significant for the most part. The shock should lower inflation and, through the Taylor rule, lower the nominal interest rate, but the error bands are wide, so the estimates are insignificant.[13]

[13]The point estimates of inflation responses show that there is a temporary, although statistically insignificant, rise in inflation following a positive TFP shock. This result comes from the endogenous responses of the nominal interest rate. In particular, the monetary policy rule (25) implies that the nominal

The neutral technology shock leads to a statistically significant decline in hours worked. The decline in hours here, however, is not a direct consequence of price stickiness. Even with much more frequent price adjustments, we find that the positive neutral technology shock leads to a decline in hours (not reported). Instead, the investment adjustment cost (as well as the habit formation to a lesser extent) plays an important role in generating the decline in hours. If the investment adjustment cost parameter is small, we find that the model generates an increase in hours following the neutral technology shock (not reported), regardless of whether prices are sticky. Thus, our finding does not support the view that the contractionary effect of a neutral technology shock arises from the price stickiness. It is consistent with Francis and Ramey (2005), who argued that a real business-cycle model with habit persistence and investment adjustment cost can generate a decline in hours following a positive neutral technology shock.

Figure 9 reports the impulse responses following a 1-standard-deviation shock to the wage markup. An increase in the wage markup raises the households' desired real wage. The households that can adjust its nominal wage raises its nominal wage. The increase in the nominal wage raises the firms' marginal cost so that inflation rises and real aggregate demand falls. It follows that aggregate output, investment, and hours decline. Consequently, the rental price of capital and utilization rise. Through the interest-rate rule, the rise in inflation leads to an increase in the nominal interest rate. All these responses are statistically significant.

8.3 *What is a shock to capital depreciation?*

The variance decompositions indicate that the TFP shock, the wage-markup shock, and the depreciation shock are the most important sources of macroeconomic fluctuations. Both the TFP shock and the wage-markup shock are familiar to many researchers, but the capital depreciation shock is new. Given its importance in accounting for the macroeconomic fluctuations in our model, it is useful to provide economic interpretations of this shock.

Like the TFP shock or any other shocks in this class of models, the depreciation shock is of reduced form that captures some "deeper" sources of disturbances and possibly microeconomic frictions that distort intertemporal capital accumulation decisions. Greenwood, Hercowitz, and Krusell (1997) drew a mapping between investment-specific technological changes (ISTC) and fluctuations in economic depreciation (as opposed to physical depreciation) of capital. They noted that the economic depreciation rate rises when the equipment price relative to the consumption price is expected to decline in the future. As the equipment price is expected to fall, existing capital is worth less and investors have an incentive to postpone investment to future periods, leading to a contraction in current economic activity, as does our depreciation shock.

interest rate responds to deviations of output from the stochastic trend λ_t^*. A positive TFP shock raises λ_t^* and lowers detrended output. Thus, in response, the nominal interest rate falls. The stimulating effect on aggregate demand of the accommodative monetary policy counteracts the cost-saving effect of the TFP shock. With our estimated parameters, the stimulating effect dominates in the short run and thus inflation rises temporarily.

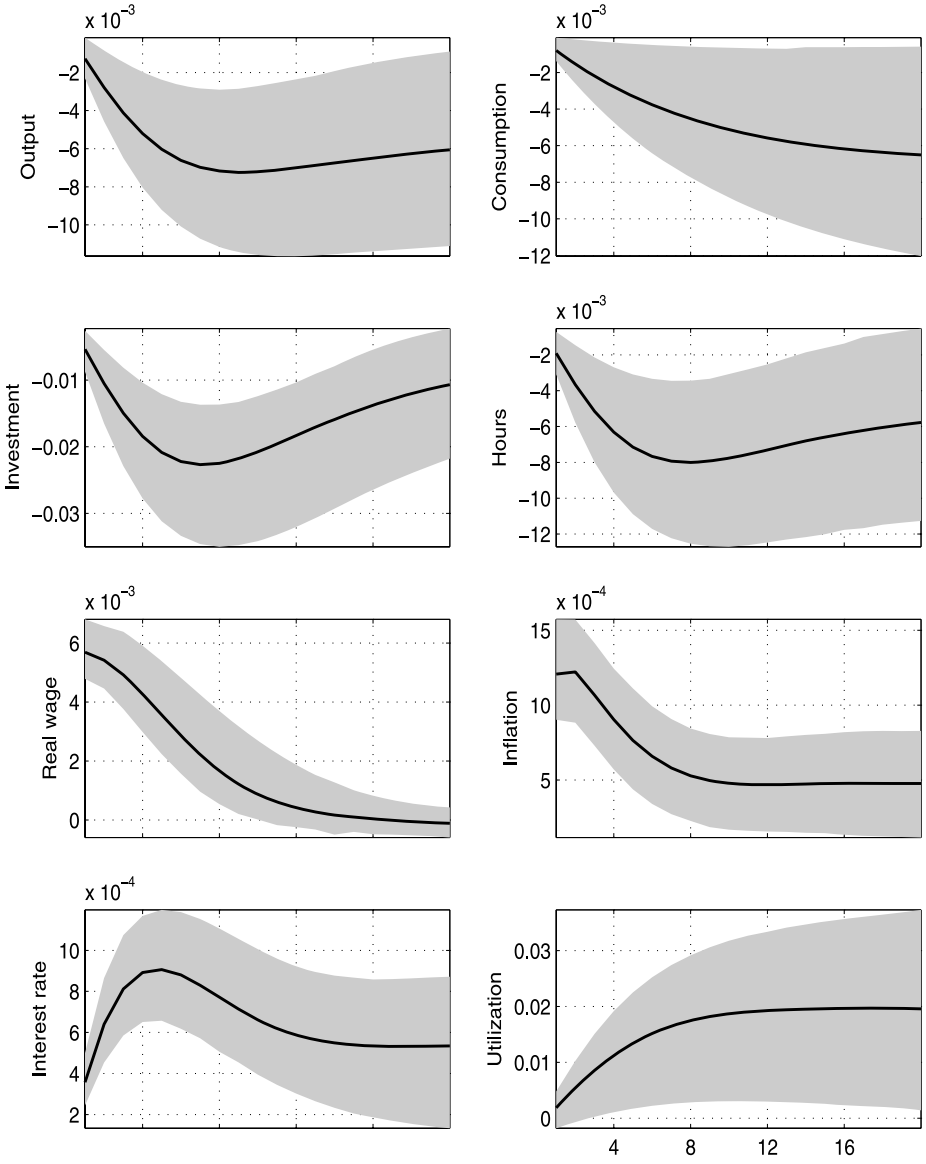

FIGURE 9. Impulse responses to a wage-markup shock in the second regime for the DSGE-2v model. The shaded area represents 0.90 probability pointwise error bands and the thick line represents the median estimate.

According to this interpretation, co-movements should be expected between ISTCs and changes in depreciation. The top panel of Figure 10 displays the historical paths of the depreciation shock series conditional on the model's estimates and the actual data of the relative price of investment.[14] These two series appear to move in tandem. To give a precise summary of the dynamic correlations between the two series, the left column of Figure 11 displays the impulse responses of ISTCs and depreciation changes following a 1-standard-deviation innovation to the ISTC. These impulse responses are estimated

[14]The smoothed estimates of the ISTC in the model is identical to the actual data of the relative price of investment since we include the relative investment price as an observable variable in our estimation.

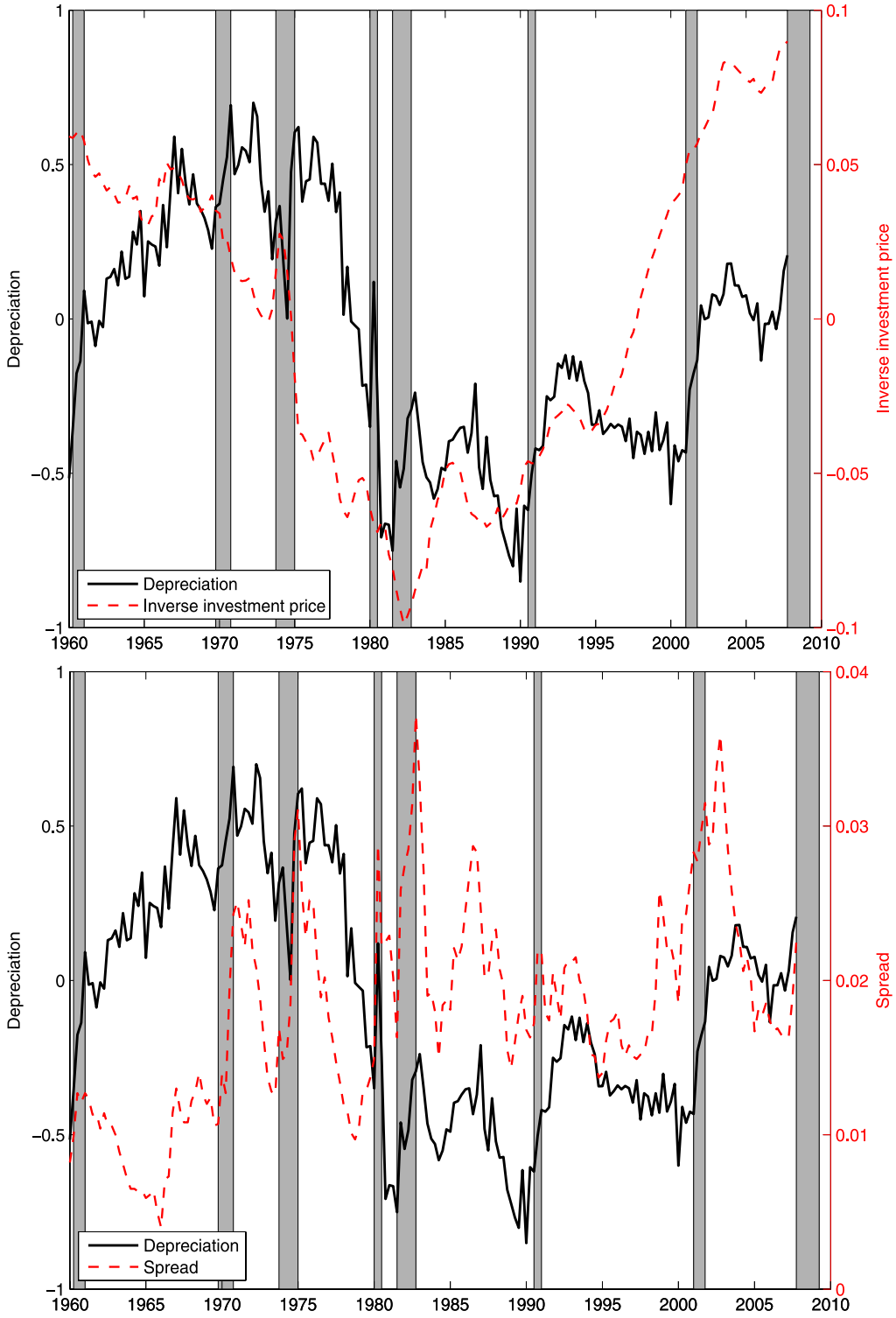

FIGURE 10. Historical paths of depreciation shocks, deviations of the ISTC from its deterministic trend, and the spread between the Baa corporate and the 10-year treasury yields. The shaded bars indicate the NBER recession dates.

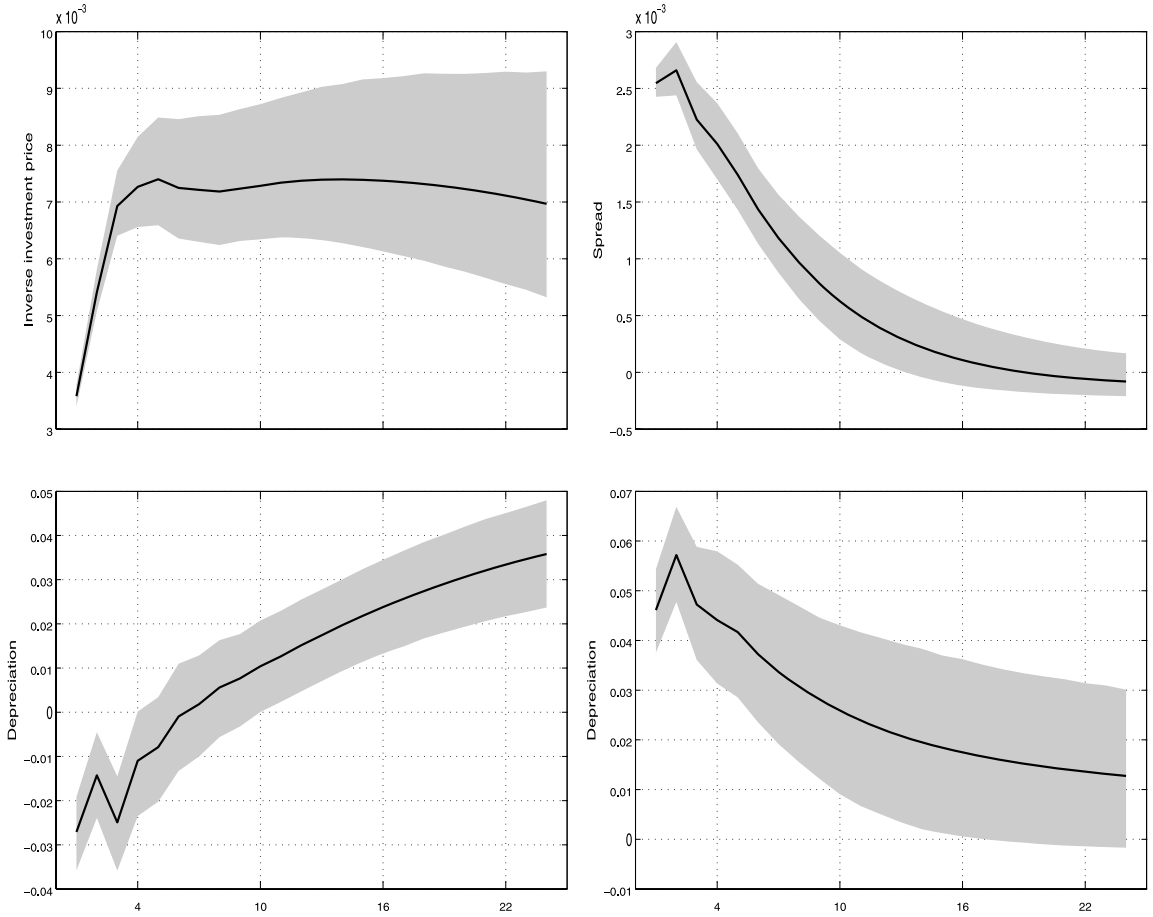

FIGURE 11. Impulse responses of the depreciation rate, the detrended ISTC, and the spread between the Baa corporate and the 10-year treasury yields. The left hand panels represent the dynamic responses of the ISTC and the depreciation rate following an innovation to the ISTC. The right hand panels represent the dynamic responses of the spread and the depreciation rate following an innovation to the spread. The shaded area represents 0.90 probability pointwise error bands and the thick line represents the median estimate.

from a recursive bivariate BVAR model with the Sims and Zha (1998) prior. The dynamic responses move together at business-cycle frequency.

There are two important aspects in which the transmission mechanism through capital depreciation is not captured by changes in the relative price of investment. First, in the short run (within the first six quarters), the dynamic responses of these two series move in opposite directions. Second and more importantly, changes in the relative price of investment are mostly driven by the deterministic trend. Once the time series of the ISTC enters the model as an observable variable, there are not enough cyclical variations left to explain the changes in the wedge of the capital Euler equation (36). Indeed, our posterior estimates show that the standard deviation of the depreciation shock is 27 times larger than the standard deviation of the detrended ISTC shock in the first regime and 50 times larger in the second regime (see Table 4).

Our alternative interpretation is that a shock to the depreciation rate represents destruction of capital and therefor resembles a shock to the quality of capital as in Justiniano, Primiceri, and Tambalotti (2011) and Gertler and Kiyotaki (2010), who interpreted their capital quality shock as representing an exogenous change in the value of capital. One possible microeconomic interpretation is that a large number of goods are produced using good-specific capital. In each period, as a fraction of goods becomes obsolete randomly, the capital used to produce those obsolete goods becomes worthless. In aggregate, the law of motion for capital would feature a depreciation shock or, similarly, a capital quality shock to reflect economic obsolescence of capital.[15]

This kind of capital destruction, as argued in Justiniano, Primiceri, and Tambalotti (2011) and Gertler and Kiyotaki (2010), is closely related to the impact of a financial shock on credit spread. The bottom panel of Figure 10 plots the historical series of depreciation shocks computed from our estimated model along with the time series of the spread between the Baa corporate bond yield and the 10-year treasury note yield. It is evident that the depreciation rate comoves more closely with the credit spread than with the ISTC. Moreover, the right column of Figure 11 reports the dynamic responses of spread and depreciation following a 1-standard-deviation innovation to the spread. Unlike the impulse responses following an ISTC shock, a shock to the credit spread drives strong comovements between depreciation and the credit spread. Thus, the data support our alternative interpretation that a shock to capital depreciation is closely related to a financial shock.

9. Conclusion

We have studied and estimated a variety of fairly large DSGE models within a unified framework to reexamine the sources of observed macroeconomic fluctuations in the post-WWII U.S. economy. We do not find strong evidence of changes in the inflation target; neither do we find support for strong nominal rigidities in prices and nominal wages. These findings are robust across a large set of regime-switching models.

Our estimation indicates that heteroskedasticity in shock disturbances is crucial for the model's empirical fit and that changes in shock variances tend to take place simultaneously rather than independently. In particular, a shock to capital depreciation, as a stand-in for economic obsolescence of capital, plays an important role as an intertemporal wedge in the capital Euler equation and generates positive comovements between consumption, investment, hours, and the real wage. The historical path of depreciation shocks moves in tandem with credit spread, suggesting that the depreciation shock in our model captures some of the effects of financial shocks that move the credit spread. The depreciation shock, along with the standard TFP shock and the wage-markup shock, is an important driving source of business-cycle fluctuations in the U.S. economy.

[15]A recent paper by Furlanetto and Seneca (2011) offered different perspectives on a depreciation shock relative to other shocks that influence capital stock or the quality of capital.

Appendix A: Detailed data description

All data were either taken directly from the Haver Analytics Database or constructed by Patrick Higgins at the Federal Reserve Bank of Atlanta. The construction methods developed or used by Patrick Higgins, available on request, are briefly described below.

The model estimation is based on quarterly time-series observations on eight U.S. aggregate variables during the sample period 1959:Q1–2007:Q4. The eight variables are real per capita GDP (Y_t^{Data}), real per capita consumption (C_t^{Data}), real per capita investment (I_t^{Data}) in capital goods, real wage (w_t^{Data}), the quarterly GDP-deflator inflation rate (π_t^{Data}), per capita hours (L_t^{Data}), the federal funds rate ($\text{FFR}_t^{\text{Data}}$), and the inverse of the relative price of investment (Q_t^{Data}).

These series are derived from the original data in the Haver Analytics Database (with the relevant data codes provided) or from the constructed data.

- $Y_t^{\text{Data}} = \frac{\text{GDPH}}{\text{POP25}-64}$.
- $C_t^{\text{Data}} = \frac{(\text{CN@USECON}+\text{CS@USECON})*100/\text{JGDP}}{\text{POP25}-64}$.
- $I_t^{\text{Data}} = \frac{(\text{CD@USECON}+\text{F@USECON})*100/\text{JGDP}}{\text{POP25}-64}$.
- $w_t^{\text{Data}} = \frac{\text{LXNFC@USECON}/100}{\text{JGDP}}$.
- $\pi_t^{\text{Data}} = \frac{\text{JGDP}_t}{\text{JGDP}_{t-1}}$.
- $L_t^{\text{Data}} = \frac{\text{LXNFH@USECON}}{\text{POP25}-64}$.
- $\text{FFR}_t^{\text{Data}} = \frac{\text{FFED@USECON}}{400}$.
- $Q_t^{\text{Data}} = \frac{\text{JGDP}}{\text{TornPriceInv4707CV}}$.

The original data, the constructed data, and their sources are described as follows.

POP25-64 Civilian noninstitutional population aged 25–64, eliminating breaks in population from 10-year censuses and post-2000 American Community Surveys using the "error of closure" method. This fairly simple method was used by the Census Bureau to get a smooth population monthly population series. This smooth series reduces the unusual influence of drastic demographic changes.

GDPH Real gross domestic product (in 2000 dollars). Source: Bureau of Economic Analysis (BEA).

CN@USECON Nominal personal consumption expenditures: nondurable goods. Source: BEA.

CS@USECON Nominal consumption expenditures: services. Source: BEA.

CD@USECON Nominal personal consumption expenditures: durable goods. Source: BEA.

F@USECON Nominal private fixed investment. Source: BEA.

JGDP Gross domestic product: chain price index (2000 = 100). Source: BEA.

LXNFC@USECON Nonfarm business sector: compensation per hour (1992 = 100). Source: Bureau of Labor Statistics (BLS).

LXNFH@USECON Nonfarm business sector: hours of all persons (1992 = 100). Source: BLS.

FFED@USECON Annualized federal funds effective rate. Source: Federal Reserve Board.

TornPriceInv4707CV Investment deflator. The Tornquist procedure is used to construct this deflator as a weighted aggregate index from the four quality-adjusted price indexes: private nonresidential structures investment, private residential investment, private nonresidential equipment and software investment, and personal consumption expenditures on durable goods. Each price index is weighted from a number of individual price series within these categories. For each individual price series from 1947 to 1983, we use Gordon's (1990) quality-adjusted price index. Following Cummins and Violante (2002), we estimate an econometric model of Gordon's price series as a function of a time trend and a few National Income and Product Accounts (NIPA) indicators (including the current and lagged values of the corresponding NIPA price series). The estimated coefficients are then used to extrapolate the quality-adjusted price index for each individual price series for the sample from 1984 to 2007. These constructed price series are annual. Denton's (1971) method is used to interpolate these annual series on a quarterly frequency. The Tornquist procedure is then used to construct each quality-adjusted price index from the appropriate interpolated quarterly price series.

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